

Exercises 9 Exponential function and equations Compound interest, exponential function

Objectives

- be able to calculate the future capital that is invested at an interest rate which is compounded annually.
- be able to treat compound interest tasks.
- be able to graph an exponential function out of its formula.
- be able to determine the formula of an exponential function out of the coordinates of two points of the graph.
- be able to treat applied tasks by means of an exponential function.

Problems

- 9.1 Compound interest at an annual rate r is paid on an initial capital C_0 .
- Assume an initial capital $C_0 = 1000.00$ CHF, and an annual interest rate $r = 2\%$. Determine the capital after one, two, three, four, and five years' time.
 - Try to develop a formula which allows you to calculate the capital C_n after n years' time for any values of C_0 , r , and n .
- 9.2 What is the future capital if 8000 CHF is invested for 10 years at 12% compounded annually?
- 9.3 What present value amounts to 10'000 CHF if it is invested for 10 years at 6% compounded annually?
- 9.4 At what interest rate, compounded annually, would 10'000 CHF have to be invested to amount to 14'071 CHF in 7 years?
- 9.5 Ms Smith wants to invest 150'000 CHF for five years. Bank A offers an interest rate of 6.5% compounded annually. Bank B offers to pay 200'000 CHF after five years. Which bank makes the better offer?
- 9.6 The purchase of Alaska cost the United States \$ 7 million in 1869. If this money had been placed in a savings account paying 6% compounded annually, how much money would be available from this investment in 2010?
- 9.7 Mary Stahley invested \$ 2500 in a 36-month certificate of deposit (CD) that earned 8.5% annual simple interest. When the CD matured, she invested the full amount in a mutual fund that had an annual growth equivalent to 18% compounded annually. How much was the mutual fund worth after 9 years?
- 9.8 A capital is invested for 4 years at 4% and for 3 more years at 6%, compounded annually. Eventually, the capital amounts to 72'000 CHF.
- Determine the initial capital.
 - What is the average interest rate with respect to the whole period of time?
- 9.9 An unknown initial capital is invested at an unknown interest rate, compounded annually. After 2 years, the capital amounts to 5'891.74 CHF, and after another 5 years the capital is 6'997.54 CHF. Determine both initial capital and interest rate.

9.10 Graph the following exponential functions in one common coordinate system:

$$f_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \rightarrow y = f_1(x) = 2^x$$

$$f_2: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \rightarrow y = f_2(x) = 0.2^x$$

$$f_3: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \rightarrow y = f_3(x) = 3 \cdot 0.5^x$$

$$f_4: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \rightarrow y = f_4(x) = -2 \cdot 3^x$$

9.11 The graph of an exponential function contains the points P and Q. Determine the formula of the exponential function.

- | | | | | | |
|----|----------------|----------------|----|------------|--------------|
| a) | P (0 1.02) | Q (1 1.0302) | b) | P (1 12) | Q (3 192) |
| c) | P (0 10'000) | Q (5 777.6) | d) | P (5 16) | Q (9 1/16) |

9.12 A house that 20 years ago was worth \$160'000 has increased in value by 4% each year because of inflation. What is its worth today?

9.13 Suppose a country has a population of 20 million and projects a growth rate of 2% per year for the next 20 years. What will the population of this country be in 10 years?

9.14 A ball is dropped from a height of 12.8 meters. It rebounds $\frac{3}{4}$ of the height from which it falls every time it hits the ground. How high will the ball bounce after it strikes the ground for the fourth time?

9.15 A machine is valued at \$10'000. The depreciation at the end of each year is 20% of its value at the beginning of the year. Find its value at the end of 4 years.

9.16 The size of a certain bacteria culture grows exponentially. At 8 a.m. and 11 a.m. the number of bacteria was 2'300 and 18'400, respectively. Determine the number of bacteria at 1.30 p.m.

9.17 In a physical experiment the number of radioactive nuclei in a certain preparation decreases exponentially. 5 hours after the start of the experiment $1.56 \cdot 10^{16}$ nuclei were counted. 3 hours later, the number has fallen to $3.05 \cdot 10^{13}$. What was the number of nuclei at the beginning of the experiment?

9.18 A capital pays interest, compounded annually. What is the interest rate such that the capital doubles in 20 years?

9.19 Suppose that the number y of otters t years after they were reintroduced into a wild and scenic river is given by

$$y = 2500 - 2490 \cdot e^{-0.1t}$$

- Find the population when the otters were introduced.
- Draw the graph of the function $f: t \rightarrow y = f(t)$.
- What is the expected upper limit of the number of otters?

- 9.20 The president of a company predicts that sales will increase after she assumes office and that the number of monthly sales will follow the curve given by

$$N = 3000 \cdot (0.2)^{0.6t}$$

where t represents the months since she assumed office.

- What will be the sales when she assumes office?
 - What will be the sales after 3 months?
 - What is the expected upper limit on sales?
- 9.21 * The consumer price index (CPI) is calculated by averaging the prices of various items after assigning a weight to each item. The following table gives the consumer price indexes for selected years from 1940 through 2002:

Year	CPI
1940	14.0
1950	24.1
1960	29.6
1970	38.8
1980	82.4
1990	130.7
2000	172.2
2002	179.9

- Find an equation that models these data, i.e. try to find the parameters a and c of the exponential function $f: x \rightarrow y = f(x) = c \cdot a^x$ (x = years after 1900, y = CPI) that fits the data.
- Use the model to predict the CPI in 2010.

Answers

- 9.1 a) $C_0 = 1000.00$ CHF $C_1 = 1020.00$ CHF $C_2 = 1040.40$ CHF
 $C_3 = 1061.21$ CHF $C_4 = 1082.43$ CHF $C_5 = 1104.08$ CHF
b) $C_n = C_0 (1 + r)^n$
- 9.2 $C_{10} = 24'846.79$ CHF
- 9.3 $C_0 = 5'583.95$ CHF
- 9.4 $r = 5\%$
- 9.5 Bank A: $C(5) = 205'513.00$ CHF
Bank B: $C(5) = 200'000.00$ CHF
- 9.6 $C_{141} = \$ 25'896$ million (rounded)
- 9.7 $\$ 13'916.24$
- 9.8 a) $C_0 = 51'675$ CHF b) $r = 4.85\%$
- 9.9 $r = 3.5\%$, $C_0 = 5'500.00$ CHF
- 9.10 ...
- 9.11 a) $y = f(x) = 1.02 \cdot 1.01^x$ b) $y = f(x) = 3 \cdot 4^x$
c) $y = f(x) = 10'000 \cdot 0.6^x$ d) $y = f(x) = 16'384 \cdot 0.25^x$
- 9.12 $\$350'580$ (rounded)
- 9.13 24.4 million (rounded)
- 9.14 4.05 meters
- 9.15 $\$4'096$
- 9.16 $104'086$
- 9.17 $5.10 \cdot 10^{20}$
- 9.18 $r = \sqrt[20]{2} - 1 = 3.5\%$ (rounded)
- 9.19 a) $y = 10$ for $t = 0$
b) ...
c) $y \rightarrow 2500$ as $t \rightarrow \infty$
- 9.20 a) $N(0) = 600$
b) $N(3) = 2119$
c) $N(t) \rightarrow 3000$ as $t \rightarrow \infty$
- 9.21 * a) $y = f(x) = 2.58 \cdot 1.043^x$
b) $y(110) = 264.79$