

### Exercises 3      **Function** **Domain, codomain, range, graph**

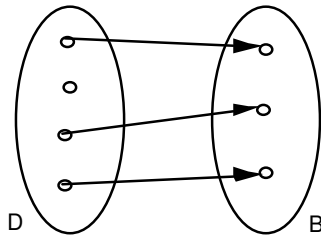
#### Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

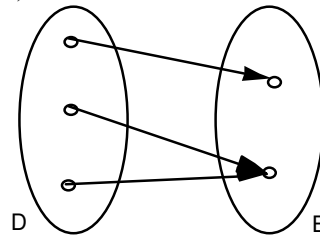
#### Problems

3.1 Which of the following relations are functions? Explain your answer.

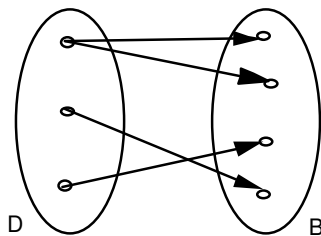
a)



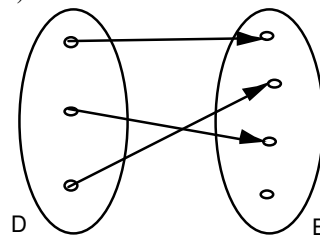
b)



c)



d)



- e)  $D =$  set of all the modules of the HTW Tourism bachelor programme  
 $B =$  set of all the HTW lecturers  
 $f: D \rightarrow B, m \rightarrow l = f(m) =$  lecturer of  $m$
- f)  $D = \{1980, 1981, \dots, 1989, 1990\}$   
 $B =$  set of all the human beings aged between 20 and 30  
 $f: D \rightarrow B, y \rightarrow p = f(y) =$  person who was born in the year  $y$
- g)  $D =$  set of all the human beings aged between 20 and 30  
 $B = \{1980, 1981, \dots, 1989, 1990\}$   
 $f: D \rightarrow B, p \rightarrow y = f(p) =$  year of birth of person  $p$
- h)  $f: \mathbb{R} \rightarrow \mathbb{R}, x \rightarrow y = f(x) = x^2$
- i)  $f: \mathbb{R}^+ \rightarrow \mathbb{R}, x \rightarrow y = f(x) =$  number whose square is  $x$
- j)  $f: \mathbb{R} \rightarrow \mathbb{R}, t \rightarrow b = f(t) =$  bank account balance at time  $t$

3.2 Determine the range E of the functions below:

- a)  $D = \{\text{January, February, March, ..., December}\}$   
 $B = \{A, B, C, ..., Z\}$   
 $f: D \rightarrow B, m \rightarrow l = f(m) = \text{initial letter of } m$
- b)  $D = \text{set of all the neighbouring countries of Switzerland}$   
 $B = \text{set of all the European cities}$   
 $c: D \rightarrow B, x \rightarrow y = c(x) = \text{capital of neighbouring country } x$
- c) function  $f$  in problem 3.1 g)
- d) function  $f$  in problem 3.1 h)

3.3 a)  $f: \mathbb{R} \rightarrow \mathbb{R}, x \rightarrow f(x) = x^3 - x$

Determine the following values:

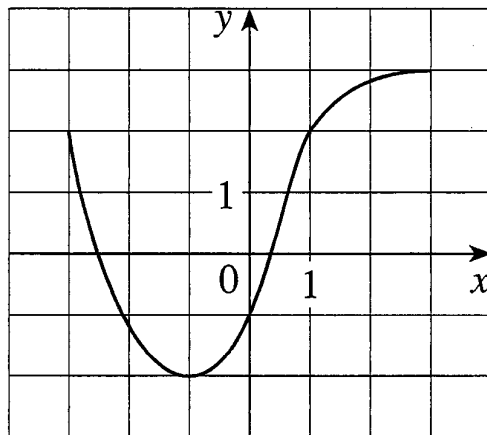
- |     |          |     |            |      |              |
|-----|----------|-----|------------|------|--------------|
| i)  | $f(1)$   | ii) | $f(-2)$    | iii) | $f(a)$       |
| iv) | $f(b^2)$ | v)  | $f(a - b)$ | vi)  | $f(x^3 - x)$ |

b)  $g: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}, x \rightarrow g(x) = \frac{x^2}{x+1}$

Determine the following values:

- |     |          |     |            |      |                                 |
|-----|----------|-----|------------|------|---------------------------------|
| i)  | $g(2)$   | ii) | $g(-3)$    | iii) | $g(a)$                          |
| iv) | $g(b^2)$ | v)  | $g(a - b)$ | vi)  | $g\left(\frac{x^2}{x+1}\right)$ |

3.4 The graph of a function  $f$  is given as follows:



- a) State the value of  $f(-1)$ .
- b) Estimate the value of  $f(2)$ .
- c) For what values of  $x$  is  $f(x) = 2$ ?
- d) Estimate the values of  $x$  such that  $f(x) = 0$ .
- e) State the domain  $D$  of  $f$ .
- f) State the range  $E$  of  $f$ .

**Answers**

- 3.1 a) no function  
 b) function  
 c) no function  
 d) function  
 e) no function  
 f) no function  
 g) function  
 h) function  
 i) no function  
 j) function
- 3.2 a)  $E = \{A, D, F, J, M, N, O, S\}$   
 b)  $E = \{\text{Berlin, Vienna, Vaduz, Rome, Paris}\}$   
 c)  $E = B$   
 d)  $E = \mathbb{R}_0^+$
- 3.3 a) i)  $f(1) = 1^3 - 1 = 0$   
 ii)  $f(-2) = (-2)^3 - (-2) = -6$   
 iii)  $f(a) = a^3 - a$   
 iv)  $f(b^2) = (b^2)^3 - b^2 = b^6 - b^2$   
 v)  $f(a - b) = (a - b)^3 - (a - b) = a^3 - 3a^2b + 3ab^2 - b^3 - a + b$   
 vi)  $f(x^3 - x) = (x^3 - x)^3 - (x^3 - x) = x^9 - 3x^7 + 3x^5 - 2x^3 + x$
- b) i)  $g(2) = \frac{2^2}{2+1} = \frac{4}{3}$   
 ii)  $g(-3) = \frac{(-3)^2}{3+1} = \frac{9}{4}$   
 iii)  $g(a) = \frac{a^2}{a+1}$   
 iv)  $g(b^2) = \frac{(b^2)^2}{b^2+1} = \frac{b^4}{b^2+1}$   
 v)  $g(a - b) = \frac{(a - b)^2}{(a - b) + 1} = \frac{a^2 - 2ab + b^2}{a - b + 1}$   
 vi)  $g\left(\frac{x^2}{x+1}\right) = \frac{\left(\frac{x^2}{x+1}\right)^2}{\left(\frac{x^2}{x+1}\right) + 1} = \frac{x^4}{x^3 + x^2 + x + 1}$
- 3.4 a)  $f(-1) = -2$   
 b)  $f(2) \approx 2.8$   
 c)  $x_1 = -3, x_2 = 1$   
 d)  $x_1 \approx -2.5, x_2 \approx 0.3$   
 e)  $D = \{x \in \mathbb{R} \mid -3 \leq x \leq 3\} = [-3, 3]$   
 f)  $E = \{y \in \mathbb{R} \mid -2 \leq y \leq 3\} = [-2, 3]$