Exercises 2 Algebra
Powers, fractions

Objective

- be able to perform basic algebraic transformations of powers and fractions.

Problems

2.1 Evaluate each expression:

a) 2^4 b) $(-2)^4$ c) -2^4

d) 3^{-4} e) $\frac{5^{23}}{5^{21}}$ f) $\left(\frac{2}{3}\right)^{-2}$

2.2 Decide whether each statement is true or false:

a) $(p+q)^2 = p^2 + q^2$ b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ c) $\sqrt{a^2+b^2} = a+b$

2.3 Simplify the following expression:

$$\frac{x^2}{x^2-4} - \frac{x+1}{x+2}$$

2.4 Decide whether each statement is true or false:

a) $\frac{1+ab}{b} = 1 + a$ b) $\frac{1}{x-y} = \frac{1}{x} - \frac{1}{y}$

2.5 Evaluate each expression:

a) $2^4 \cdot 2^3$ b) $2^4 \cdot 2^{-3}$ c) $2^4 \cdot (-2)^{-3}$

d) $(2^3)^2$ e) $(2^{-3})^2$ f) $(-2^{-3})^{-2}$

g) $\left((-2)^{-3}\right)^{-2}$ h) $-(2^{-3})^{-2}$ i) $\left(-\frac{1}{5}\right)^{-2}$

j) $\left(-\frac{3}{4}\right)^{-3}$

2.6 Simplify each expression:

a) $a^3 \cdot a^2$ b) $5^{n-1} \cdot 5^4$ c) $7^{n+1} \cdot 7^{n-1}$

d) $a^{x+5} : (a^x \cdot a^5)$ e) $(2a^3 \cdot 3a^2)^2$ f) $(a^2b)^{25} \cdot (ab^4)^{25}$

g) $\frac{10a^{-3}}{5a^{-2}}\, 2a^3$

2.7 Simplify each fraction:

a) $\frac{24a^2bc^2}{56abc}$ b) $\frac{uw}{uv+uw}$ c) $\frac{n^3-n}{n^3+n^2}$

2.8 Simplify and rewrite the expression with a single fraction:

a) $\frac{1}{m+1} + \frac{m}{m+1}$

b) $\frac{2p}{15q} + \frac{8p}{9q}$

c) $\frac{1}{r^2} - \frac{1}{r^3}$

d) $d - \frac{nd-2}{n}$

e) $\frac{t+7}{3t-6} - \frac{t+4}{t^2-2t}$

f) $\frac{d-1}{18d} \cdot \frac{12d^2}{1-d}$

g) $\frac{\frac{u}{v}}{x}$

h) $\frac{x}{\frac{u}{v}}$

i) $\frac{2e-6f}{\frac{3e^2-9ef}{2f}}$

j) $\frac{\frac{n}{n^2-1}}{\frac{1}{n+1} - \frac{1}{n-1}}$

Answers

2.1 a) 16 b) 16 c) - 16
d) $\frac{1}{81}$ e) 25 f) $\frac{9}{4}$

2.2 a) false b) true c) false

2.3 $\frac{1}{x-2}$

2.4 a) false b) false

2.5 a) 128 b) 2 c) -2
d) 64 e) $\frac{1}{64}$ f) 64
g) 64 h) -64 i) 25
j) $-\frac{64}{27}$

2.6 a) a^5 b) 5^{n+3} c) 7^{2n}
d) 1 e) $36a^{10}$ f) $a^{75}\, b^{125}$
g) $4a^2$

2.7 a) $\frac{3ac}{7}$ b) $\frac{w}{v+w}$ c) $\frac{n-1}{n}$

2.8 a) 1 b) $\frac{46p}{45q}$ c) $\frac{r-1}{r^3}$
d) $\frac{2}{n}$ e) $\frac{t+6}{3t}$ f) $-\frac{2d}{3}$
g) $\frac{u}{vx}$ h) $\frac{vx}{u}$ i) $\frac{4f}{3e}$
j) $-\frac{n}{2}$