Review exercises

Problems

R1.1 The sets U, A, and B are given as follows:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {1, 2, 3, 9}
B = {1, 3, 5, 6, 7, 8, 9}

Find the elements of the sets below:

a) $A \cap B$

b) $A \quad (U \setminus B)$

c) $(U \setminus A) \cap B$

d) $U \setminus ((U \setminus A) \cap B)$

R1.2 Simplify each of the following expressions:

a) $x^5 \cdot x^{-7}$

b) $\dfrac{x^8}{x^{-2}}$

c) $(-y^{-3})^{-2}$

d) $(x - y)(x^2 + xy + y^2)$

e) $\dfrac{4x^2y^3 - 6x^3y^4}{2x^2y^2 - 3xy^3}$

f) $\dfrac{x - 1 - \dfrac{x - 1}{x}}{\dfrac{1}{x - 1} + 1}$

R1.3 Which of the following relations are functions? Explain your answers.

a) $f_1 : \mathbb{R}_0^+ \to \mathbb{R}^+, x \to y = f_1(x) = \sqrt{x}$

b) $f_2 : \{2,3,4,...\} \to \mathbb{N}, x \to y = f_2(x) = x - 1$

c) D = Set of all Swiss cantons
B = Set of all Swiss towns and cities
$f_3 : D \to B, x \to y = f_3(x) =$ capital of x

d) $f_4 : \{x \quad \mathbb{R} \mid x \geq 3\} \to R, x \to y = f_4(x) = \dfrac{1}{x^2 - 9}$

e) $f_5 : \mathbb{R}_0^+ \to \mathbb{R}, x \to y = f_5(x) = \log_a(x)$

R1.4 Determine the largest possible domain D and the corresponding range E of the following functions:

a) $f_1 : D \to B, x \to y = f_1(x) = \sqrt{9 - x}$

b) $f_2 : D \to B, x \to y = f_2(x) = \dfrac{1}{1 - x^2}$

R1.5 If $f(x) = 9x - x^2$, find ...

a) ... f(0).

b) ... f(-3).

c) ... $\frac{f(x + h) - f(x)}{h}$ and simplify.

R1.6 Solve the equations below:

a) $3x - 8 = 23$

b) $\frac{6}{3x - 5} = \frac{6}{2x + 3}$

c) $\frac{2x + 5}{x + 7} = \frac{1}{3} + \frac{x - 11}{2x + 14}$

R1.7 Solve the following equations for x, and determine the solution sets.
Take into account that the parameters a and p can be any real numbers.

a) $ax = 60$

b) $(p - 1)px = p^2 - 1$

R1.8 Solve each system of equations:

a) $2x + y = 19$
$x - 2y = 12$

b) $6x + 3y = 1$
$y = - 2x + 1$

R1.9 Find the formula of the linear function whose graph ...

a) ... has slope 4 and intercept 2.

b) ... passes through (-2|1) and has slope $\frac{2}{5}$.

c) ... passes through (-2|7) and (6|-4).

d) ... passes through (1|6) and is parallel to $y = 4x - 6$.

R1.10 A certain product has the following supply and demand functions:

$p = f_s(q) = (4q + 5)$ CHF
$p = f_d(q) = (- 2q + 81)$ CHF

a) If the price is \$53, how many units are supplied and how many are demanded?

b) Find both the equilibrium quantity and the equilibrium price.

R1.11 The total cost and total revenue for a certain product are given by the following:

$C(x) = (38.80x + 4500)$ CHF
$R(x) = 61.30x$ CHF

a) Determine the fix costs.

b) Determine the variable costs for producing 10 units.

c) Find the number of units required to break even.

R1.12 The supply function and the demand function for a product are linear and are determined by the tables that follow. Find the quantity and price that will give market equilibrium.

Supply function		Demand function	
Price	Quantity	Price	Quantity
\$100	200	\$200	200
\$200	400	\$100	400
\$300	600	\$0	600

R1.13 Find the solutions to each equation:

a) $4x - 3x^2 = 0$

b) $3x^2 - 6x = 9$

c) $4x^2 + 25 = 0$

d) $\frac{1}{x} + 2x = \frac{1}{3} + \frac{x + 1}{x}$

e) $\frac{x - 4}{x - 5} = \frac{30 - x^2}{x^2 - 5x}$

R1.14 Find the formula of the quadratic function whose graph ...

a) ... has the vertex (2|4) and passes through (3|3).

b) ... passes through (-3|-3), (0|3), and (3|0).

R1.15 The supply function for a product is given by $p = q^2 + 300$, and the demand is given by $p + q = 410$. Find the equilibrium quantity and price.

R1.16 If total costs for a product are given by $C(x) = 1760 + 8x + 0.6x^2$ and total revenues are given by $R(x) = 100x - 0.4x^2$, find the break-even points.

R1.17 Consider the following function f:

$$\begin{array}{lll} f: & \mathbb{R} & \to \mathbb{R} \\ & x & \to y = f(x) = (k - x)(x - 2) - k(x^2 - 2) - 1 \quad (k \in \mathbb{R}) \end{array}$$

Determine the value(s) of k such that the graph of f and the x-axis have exactly one point in common.

R1.18 The functions f and g are defined as follows:

f: $\mathbb{R} \to \mathbb{R},\ x \to y = f(x) = 2x^2+4x+1$

g: $\mathbb{R} \to \mathbb{R},\ x \to y = g(x) = ax + \frac{1}{2} \quad (a \in \mathbb{R} \setminus \{0\})$

Determine the value(s) of a such that the graphs of f and g touch.

R1.19 Find the formula of the exponential function whose graph passes through P and Q.

a) P(0|1) Q(2|9)

b) P(1|20) Q(2|100)

R1.20 Evaluate each logarithm without using a calculator:

a) $\log_5(1)$

b) $\log_2(8)$

c) $\log_3\left(\frac{1}{3}\right)$

d) $\log_3(3^8)$

e) $e^{\ln(5)}$

f) $10^{\log(3.15)}$

R1.21 Solve each equation:

a) $6^{4x} = 46'656$

b) $8000 = 250 \cdot 1.07^x$

c) $312 = 300 + 300\, e^{-0.08x}$

R1.22 If \$8000 is borrowed at 12% simple interest for 3 years, what is the future value of the loan at the end of the 3 years?

R1.23 Mary Toy borrowed \$2000 from her parents and repaid them \$2100 after 9 months. What simple interest rate did she pay?

R1.24 How much summer earnings must a college student deposit on August 31 in order to have \$3000 for tuition and fees on December 31 of the same year, if the investement earns 6% simple interest?

R1.25 If \$1000 is invested for 4 years at 8%, compounded quarterly, how much interest will be earned?

R1.26 How much must one invest now in order to have \$18'000 in 4 years if the investement earns 5.4%, compounded monthly?

R1.27 In 1990 an African country had a population of 4.5 million. The population has been increasing at 4% per year. What will the population be in 2010 if the growth factor does not change?

R1.28 A company wants to have \$250'000 available in 4 1/2 years for new construction. How much must be deposited at the beginning of each quarter to reach this goal if the investement earns 10.2%, compounded quarterly?

R1.29 A retirement account that earns 6.8%, compounded semiannually, contains \$488'000. How long can \$40'000 be withdrawn at the end of each half-year until the account balance is \$0?

R1.30 Three years from now, a couple plan to spend 4 months travelling in China, Japan, and Southeast Asia. When they take their trip, they would like to withdraw $5000 at the beginning of each month to cover their expenses for that month. Starting now, how much must they deposit at the beginning of each month for the next 3 years so that the account will provide the money they want while they are travelling? Assume that such an account pays 6.6%, compounded monthly.

R1.31 Mr. S is obligated to pay 25'000 CHF at the end of each of the following 8 years to his divorced wife. As a result of a personal profit in his company, he is able to pay the whole sum at the end of the first year (instead of making 8 payments at the end of each year). What amount of money does he have to pay at the end of the first year if the annual interest rate has been fixed at 4.5%?

R1.32 Mr. P is thinking about an investement for his retirement. He would like to withdraw 8000 CHF from an account at the end of each year for 15 years starting at the end of the year in which he turns 60. He assumes an annual interest rate of 2.5%.

a) He wants to save the money by making 30 constant payments at the end of each year until turning 55. How much must he pay in each year, if his banks pays him 3%, compounded annually?

b) Mr. P has won 40'000 CHF in a lottery! Would this amount be sufficient for his retirement scheme if he pays the money in at the end of the year in which he turns 25? Assume the same interest rate as in a).

Answers

R1.1 a) $A \cap B = \{1, 3, 9\}$

b) $A \quad (U \setminus B) = \{1, 2, 3, 4, 9, 10\}$

c) $(U \setminus A) \cap B = \{5, 6, 7, 8\}$

d) $U \setminus ((U \setminus A) \cap B) = \{1, 2, 3, 4, 9, 10\}$

R1.2 a) $\frac{1}{x^2}$ b) x^{10} c) y^6

d) $x^3 - y^3$ e) $\frac{2xy(2 - 3xy)}{2x - 3y}$ f) $\frac{(x - 1)^3}{x^2}$

R1.3 a) no function (no element in $\mathbb{R}^+$ is assigned to x = 0)

b) function

c) function

d) no function (f not defined for x = 3)

e) function

R1.4 a) $9 - x \geq 0$
$D = \{x \quad \mathbb{R} \mid x \leq 9\}$
$E = \mathbb{R}_0^+$

b) $1 - x^2 \neq 0$
$D = \mathbb{R} \setminus \{-1, 1\}$
$E = \mathbb{R} \setminus \{y \quad \mathbb{R} \mid 0 \leq y < 1\}$

R1.5 a) $f(0) = 0$ b) $f(-3) = -36$ c) $\frac{f(x + h) - f(x)}{h} = 9 - 2x - h$

R1.6 a) $S = \left\{\frac{31}{3}\right\}$ b) $S = \{8\}$ c) $S = \{-7\}$

R1.7 a) dividing by a only allowed if $a \neq 0$

if a = 0:	no solution	$S = \{\ \}$
if $a \neq 0$:	$x = \frac{60}{a}$	$S = \left\{\frac{60}{a}\right\}$

b) dividing by p only allowed if $p \neq 0$
dividing by (p - 1) only allowed if $(p - 1) \neq 0$

if p = 0:	no solution	$S = \{\ \}$
if p = 1:	$x \quad \mathbb{R}$	$S = \mathbb{R}$
if $p \neq 0$ and $p \neq 1$:	$x = \frac{p + 1}{p}$	$S = \left\{\frac{p + 1}{p}\right\}$

R1.8 a) $(x, y) = (10, -1)$
$S = \{(10, -1)\}$

b) no solution
$S = \{\ \}$

R1.9 a) $y = f(x) = 4x + 2$ b) $y = f(x) = \frac{2}{5}x + \frac{9}{5}$ c) $y = f(x) = -\frac{11}{8}x + \frac{17}{4}$

d) $y = f(x) = 4x + 2$

R1.10 a) 12 supplied, 14 demanded

b) $f_s(q) = f_d(q)$ for $q = \frac{38}{3} = 12.3... \notin \mathbb{N}$ no equilibrium

R1.11 a) 4500 CHF b) 388 CHF c) $x = 200$

R1.12 $q = 300$, $p = \$150$

R1.13 a) $S = \{0, 4/3\}$ b) $S = \{-1, 3\}$ c) $S = \{\ \}$

d) $S = \{2/3\}$ e) $S = \{-3\}$

R1.14 a) $y = f(x) = -(x - 2)^2 + 4$

b) $y = f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$

R1.15 $q = 10$, $p = 400$

R1.16 $x_1 = 46 + 2\sqrt{89}$ $x_2 = 46 - 2\sqrt{89}$

R1.17 The equation $f(x) = 0$ must have exactly one solution.
$k_1 = 0$, $k_2 = -1$

R1.18 The equation $f(x) = g(x)$ must have exactly one solution.
$a_1 = 2$, $a_2 = 6$

R1.19 a) $y = f(x) = 3^x$ b) $y = f(x) = 4 \cdot 5^x$

R1.20 a) 0 b) 3 c) - 1

d) 8 e) 5 f) 3.15

R1.21 a) $x = 1.5$ b) $x = 51.22...$ c) $x = 40.23...$

R1.22 $C_3 = \$10'880$

R1.23 $r = 6\ 2/3\%$

R1.24 C_0 = \$2941.18

R1.25 \$372.79

R1.26 C_0 = \$14'510.26

R1.27 9.86 million

R1.28 p = \$10'841.24

R1.29 n = 16.02... → 16 half-years = 8 years

R1.30 p = \$497.04

R1.31 172'317.53 CHF (rounded up)

R1.32 a) p = 1795.93 CHF

b) yes