

Exercises 11 Exponential function and equations Compound interest, nominal/effective annual interest rate

Objectives

- be able to calculate the future capital that is invested at an interest rate which is compounded more than once per year.
- know and understand the terms "nominal annual interest rate" and "effective annual interest rate".
- be able to treat specific compound interest tasks.

Problems

- 11.1 An initial capital $C_0 = 1000$ CHF is invested at a nominal annual interest rate $r = 10\%$, compounded ...
- a) ... quarterly.
 - i) Determine the capitals C_1 , C_2 , and C_3 , after one, two, and three years respectively.
 - ii) Determine the effective annual interest rate r^* .
 - b) ... monthly.
 - i) Determine the capitals C_1 , C_2 , and C_3 , after one, two, and three years respectively.
 - ii) Determine the effective annual interest rate r^* .
- 11.2 Determine the effective annual interest rate for a nominal annual interest rate of 6% , compounded ...
- a) ... annually.
 - b) ... semiannually.
 - c) ... quarterly.
 - d) ... monthly.
 - e) ... daily (1 year = 360 days).
- 11.3 What is the future value if $\$3200$ is invested for 5 years at 8% compounded quarterly?
- 11.4 Find the interest that will be earned if $\$10'000$ is invested for 3 years at 9% compounded monthly.
- 11.5 What amount of money do parents need to deposit in an account earning 10% , compounded monthly, so that it will grow to $\$40'000$ for their son's college tuition in 18 years?
- 11.6 An initial capital of 1000 CHF amounts to 1500 CHF if it is invested for 10 years at an unknown annual interest rate, compounded quarterly.
- Determine the ...
- a) ... nominal annual interest rate.
 - b) ... effective annual interest rate.
- 11.7 How long (in months) would a capital have to be invested at 6% , compounded monthly, to double its value?

- 11.8 Ms P. wants to invest 100'000 CHF. Her bank makes two offers:
- A effective annual interest rate of 8.5%
 - B nominal annual interest rate of 8%, compounded monthly
- Which offer is better, offer A or offer B?
- 11.9 How long (in years) would 1000 CHF have to be invested at 2.5%, compounded daily, to earn 250 CHF interest?
- 11.10 At what nominal rate, compounded quarterly, would \$20'000 have to be invested to amount to \$26'425.82 in 7 years?
- 11.11 A couple needs \$15'000 as a down payment for a home. If they invest the \$10'000 they have at 8% compounded quarterly, how long will it take for the money to grow into \$15'000?

Answers

- 11.1 a) i) $C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$
 $C_1 = 1000 \left(1 + \frac{0.1}{4}\right)^{4 \cdot 1}$ CHF = 1103.81 CHF (rounded)
 $C_2 = 1000 \left(1 + \frac{0.1}{4}\right)^{4 \cdot 2}$ CHF = 1218.40 CHF (rounded)
 $C_3 = 1000 \left(1 + \frac{0.1}{4}\right)^{4 \cdot 3}$ CHF = 1344.89 CHF (rounded)
- ii) $r^* = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 0.1047 = 10.38\%$ (rounded)
- b) i) $C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$
 $C_1 = 1000 \left(1 + \frac{0.1}{12}\right)^{12 \cdot 1}$ CHF = 1104.71 CHF (rounded)
 $C_2 = 1000 \left(1 + \frac{0.1}{12}\right)^{12 \cdot 2}$ CHF = 1220.39 CHF (rounded)
 $C_3 = 1000 \left(1 + \frac{0.1}{12}\right)^{12 \cdot 3}$ CHF = 1348.18 CHF (rounded)
- ii) $r^* = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.1047 = 10.47\%$ (rounded)
- 11.2 $r^* = \left(1 + \frac{r}{m}\right)^m - 1$ $r = 6\% = 0.06$
- a) $m = 1$ $r^* = 6\%$
 b) $m = 2$ $r^* = 6.09\%$
 c) $m = 4$ $r^* = 6.136\%$ (rounded)
 d) $m = 12$ $r^* = 6.168\%$ (rounded)
 e) $m = 360$ $r^* = 6.183\%$ (rounded)
- 11.3 $C_5 = \$4755.03$ (rounded)
- 11.4 $\$3086.45$ (rounded)
- 11.5 $C_0 = \$6661.46$ (rounded)
- 11.6 a) $r = 4.08\%$ (rounded)
 b) $r^* = 4.14\%$ (rounded)
- 11.7 $n = 11.58\dots$
 $mn = 138.98\dots$ 139 months = 11 years 7 months

11.8 $r^*(A) = 8.5\%$
 $r^*(B) = 8.3\% < 8.5\%$

Offer A is better than offer B

11.9 $n = 8.92\dots$ 9 years

11.10 $r = 4\%$

11.11 $n = 5.11\dots$
 $mn = 20.47\dots$ 21 quarters = 5 years 3 months