

## Exercises 9                      Exponential function and equations Compound interest, exponential function

### Objectives

- be able to calculate the future capital that is invested at an interest rate which is compounded annually.
- be able to treat compound interest tasks.
- be able to graph an exponential function out of its formula.
- be able to determine the formula of an exponential function out of the coordinates of two points of the graph.
- be able to treat applied tasks by means of an exponential function.

### Problems

- 9.1      Compound interest at an annual rate  $r$  is paid on an initial capital  $C_0$ .
- a)      Assume an initial capital  $C_0 = 1000.00$  CHF, and an annual interest rate  $r = 2\%$ . Determine the capital after one, two, three, four, and five years' time.
  - b)      Try to develop a formula which allows you to calculate the capital  $C_n$  after  $n$  years' time for any values of  $C_0$ ,  $r$ , and  $n$ .
- 9.2      What is the future capital if 8000 CHF is invested for 10 years at 12% compounded annually?
- 9.3      What present value amounts to 10'000 CHF if it is invested for 10 years at 6% compounded annually?
- 9.4      At what interest rate, compounded annually, would 10'000 CHF have to be invested to amount to 14'071 CHF in 7 years?
- 9.5      Ms Smith wants to invest 150'000 CHF for five years. Bank A offers an interest rate of 6.5% compounded annually. Bank B offers to pay 200'000 CHF after five years. Which bank makes the better offer?
- 9.6      The purchase of Alaska cost the United States \$ 7 million in 1869. If this money had been placed in a savings account paying 6% compounded annually, how much money would be available from this investment in 2010?
- 9.7      Mary Stahley invested \$ 2500 in a 36-month certificate of deposit (CD) that earned 8.5% annual simple interest. When the CD matured, she invested the full amount in a mutual fund that had an annual growth equivalent to 18% compounded annually. How much was the mutual fund worth after 9 years?
- 9.8      A capital is invested for 4 years at 4% and for 3 more years at 6%, compounded annually. Eventually, the capital amounts to 72'000 CHF.
- a)      Determine the initial capital.
  - b)      What is the average interest rate with respect to the whole period of time?
- 9.9      An unknown initial capital is invested at an unknown interest rate, compounded annually. After 2 years, the capital amounts to 5'891.74 CHF, and after another 5 years the capital is 6'997.54 CHF. Determine both initial capital and interest rate.

9.10 Graph the following exponential functions in one common coordinate system:

$$f_1: \begin{array}{l} \mathbb{R} \\ x \end{array} \quad \begin{array}{l} \mathbb{R} \\ y = f_1(x) = 2^x \end{array}$$

$$f_2: \begin{array}{l} \mathbb{R} \\ x \end{array} \quad \begin{array}{l} \mathbb{R} \\ y = f_2(x) = 0.2^x \end{array}$$

$$f_3: \begin{array}{l} \mathbb{R} \\ x \end{array} \quad \begin{array}{l} \mathbb{R} \\ y = f_3(x) = 3 \cdot 0.5^x \end{array}$$

$$f_4: \begin{array}{l} \mathbb{R} \\ x \end{array} \quad \begin{array}{l} \mathbb{R} \\ y = f_4(x) = -2 \cdot 3^x \end{array}$$

9.11 The graph of an exponential function contains the points P and Q.  
Determine the formula of the exponential function.

- a) P (0 | 1.02)      Q (1 | 1.0302)
- b) P (1 | 12)      Q (3 | 192)
- c) P (0 | 10'000)      Q (5 | 77.76)
- d) P (5 | 16)      Q (9 | 1/16)

9.12 A house that 20 years ago was worth \$160'000 has increased in value by 4% each year because of inflation. What is its worth today?

9.13 Suppose a country has a population of 20 million and projects a growth rate of 2% per year for the next 20 years. What will the population of this country be in 10 years?

9.14 A ball is dropped from a height of 12.8 meters. It rebounds  $\frac{3}{4}$  of the height from which it falls every time it hits the ground. How high will the ball bounce after it strikes the ground for the fourth time?

9.15 A machine is valued at \$10'000. The depreciation at the end of each year is 20% of its value at the beginning of the year. Find its value at the end of 4 years.

9.16 The size of a certain bacteria culture grows exponentially. At 8 a.m. and 11 a.m. the number of bacteria was 2'300 and 18'400, respectively. Determine the number of bacteria at 1.30 p.m.

9.17 In a physical experiment the number of radioactive nuclei in a certain preparation decreases exponentially. 5 hours after the start of the experiment  $1.56 \cdot 10^{16}$  nuclei were counted. 3 hours later, the number has fallen to  $3.05 \cdot 10^{13}$ . What was the number of nuclei at the beginning of the experiment?

9.18 A capital pays interest, compounded annually. What is the interest rate such that the capital doubles in 20 years?

**Answers**

9.1 a)  $C_0 = 1000.00 \text{ CHF}$      $C_1 = 1020.00 \text{ CHF}$      $C_2 = 1040.40 \text{ CHF}$   
 $C_3 = 1061.21 \text{ CHF}$      $C_4 = 1082.43 \text{ CHF}$      $C_5 = 1104.08 \text{ CHF}$   
b)  $C_n = C_0(1 + r)^n$

9.2  $C_{10} = 24'846.79 \text{ CHF}$

9.3  $C_0 = 5'583.95 \text{ CHF}$

9.4  $r = 5\%$

9.5 Bank A:  $C(5) = 205'513.00 \text{ CHF}$   
Bank B:  $C(5) = 200'000.00 \text{ CHF}$

9.6  $C_{141} = \$ 25'896 \text{ million (rounded)}$

9.7  $\$ 13'916.24$

9.8 a)  $C_0 = 51'675 \text{ CHF}$                       b)  $r = 4.85\%$

9.9  $r = 3.5\%$ ,  $C_0 = 5'500.00 \text{ CHF}$

9.10 ...

9.11 a)  $y = f(x) = 1.02 \cdot 1.01^x$                       b)  $y = f(x) = 3 \cdot 4^x$   
c)  $y = f(x) = 10'000 \cdot 0.6^x$                       d)  $y = f(x) = 16'384 \cdot 0.25^x$

9.12  $\$350'850 \text{ (rounded)}$

9.13  $24.4 \text{ million (rounded)}$

9.14  $4.05 \text{ meters}$

9.15  $\$4'096$

9.16  $104'086$

9.17  $5.10 \cdot 10^{20}$

9.18  $r = \sqrt[20]{2} - 1 = 3.5\% \text{ (rounded)}$