Exercises 8 Quadratic function and equations
Quadratic function/equations, supply, demand, market equilibrium

Objectives

- be able to solve special quadratic equations without applying the quadratic formula.
- be able to solve a quadratic equation by applying the quadratic formula.
- be able to solve a quadratic equation containing a parameter.
- be able to determine the vertex form of a quadratic function out of the coordinates of the vertex and the coordinates of another point of the corresponding parabola.
- be able to determine the general form of a quadratic function out of the coordinates of three points of the corresponding parabola.
- be able to treat applied tasks in economics by means of quadratic equations or systems of quadratic equations.

Problems

8.1 Solve the quadratic equations below without using the quadratic formula.
State the solution set for each equation.

a) $(x + 2)(x + 5) = 0$ b) $(x - 8)(5x - 9) = 0$

c) $x^2 - 3x = 0$ d) $x^2 + 7x = 0$

e) $4x^2 - 9 = 0$ f) $100x^2 - 1 = 0$

g) $(3x - 2)(4x + 1) = 0$ h) $4x^2 + 5x = 0$

i) $3x^2 = 27$ j) $x^2 = x$

8.2 Solve the quadratic equations below. State the solution set for each equation.

a) $(7 + x)(7 - x) = (3x + 2)^2 - (2x + 3)^2$ b) $(x - 3)(2x - 7) = 1$

c) $\frac{8}{x^2 - 4} + \frac{2}{2 - x} = 3x - 1$ d) $\frac{x - 4}{x - 5} = \frac{30 - x^2}{x^2 - 5x}$

e) $\frac{x^2 - x - 2}{2 - x} = 1$ f) $\frac{x^2 - 4}{x^2 - 4} = 1$

8.3 Determine the value(s) of the parameter b such that the quadratic equation has exactly one solution.
State this solution:

a) $2x^2 = 3x - b$

b) $x^2 + bx + b = - 3$

8.4 Solve the following equations for x. Take into account that the parameter b can have any real value.

a) $x^2 + x + b = 0$

b) $- bx = 1 + 4x^2$

8.5 A parabola has the vertex V and contains the point P.
Determine the formula of the corresponding quadratic function both in the vertex and in the general form.

a) V(2|4) P(-1|7)

b) V(1|-8) P(2|-7)

8.6 A parabola contains the three points P, Q, and R.
Determine the formula of the corresponding quadratic function in the general form.

a)	P(-4|8)	Q(0|0)	R(10|15)
b)	P(1|-1)	Q(2|4)	R(4|8)

8.7 Find the equilibrium quantity and equilibrium price of a commodity for the given supply and demand functions f_s and f_d:

a) supply $p = f_s(q) = \frac{1}{4} q^2 + 10$
demand $p = f_d(q) = 86 - 6q - 3q^2$

b) supply $p = f_s(q) = q^2 + 8q + 16$
demand $p = f_d(q) = - 3q^2 + 6q + 436$

8.8 The total costs and the total revenues for a company are given by

$C(x) = 2000 + 40x + x^2$
$R(x) = 130x$

Find the break-even points.

8.9 The costs C(x) for producing x items and the revenue R(x) for selling x items are given below.
How many items are to be produced and sold in order to achieve a profit of 200 CHF?

$C(x) = (x^2 + 100x + 80)$ CHF
$R(x) = (160x - 2x^2)$ CHF

Answers

8.1 a) S = {-5, -2} b) S = {9/5, 8}

c) S = {0, 3} d) S = {-7, 0}

e) S = {-3/2, 3/2} f) S = {-1/10, 1/10}

g) S = {-1/4, 2/3} h) S = {-5/4, 0}

i) S = {-3, 3} j) S = {0, 1}

8.2 a) S = {-3, 3} b) S = {5/2, 4}

c) S = {-5/3, 0} d) S = {-3}

e) S = {-2} f) S = $\mathbb{R}\backslash\{-2, 2\}$

8.3 a) $b = \frac{9}{8}$ $x = \frac{3}{4}$

b) $b_1 = -2$ $x = 1$
$b_2 = 6$ $x = -3$

8.4 a) $b < \frac{1}{4}$ $x_{1,2} = \frac{-1 \pm \sqrt{1 - 4b}}{2}$ 2 solutions

$b = \frac{1}{4}$ $x = -\frac{1}{2}$ 1 solution

$b > \frac{1}{4}$ S = { } no solution

b) $|b| > 4$ $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 16}}{8}$ 2 solutions

$b = \pm 4$ $x = -\frac{b}{8}$ 1 solution

$|b| < 4$ S = { } no solution

8.5 a) $y = f(x) = \frac{1}{3}(x - 2)^2 + 4 = \frac{1}{3}x^2 - \frac{4}{3}x + \frac{16}{3}$

b) $y = f(x) = (x - 1)^2 - 8 = x^2 - 2x - 7$

8.6 a) $y = f(x) = \frac{1}{4}x^2 - x$

b) $y = f(x) = -x^2 + 8x - 8$

8.7 a) at market equilibrium: q = 4, p = 14

b) at market equilibrium: q = 10, p = 196

8.8 $x_1 = 40$, $x_2 = 50$

8.9 profit $P(x) = R(x) - C(x) = -3x^2 + 60x - 80 \overset{!}{=} 200$

S = {7.41..., 12.58...}

7 or 13 items