

Exercises 1 Sets Intersection, complement, union, number sets

Objectives

- understand what a set, an element of a set, an intersection, a union, a complement is.
- know the set of real numbers, rational numbers, integers, natural numbers.
- be able to perform basic set operations.

Problems

1.1 Look at the sets A, B, and C:

A = Set of all the cities of the world

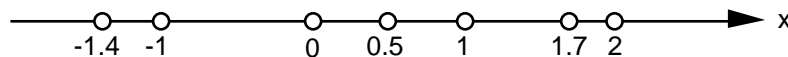
B = Set of all the European cities

C = Set of all the coastal cities of the world

Find at least four elements of the following sets:

- | | |
|--------------------|-----------------------------|
| a) $B \cap C$ | b) $B \setminus C$ |
| c) $C \setminus B$ | d) $A \setminus (B \cap C)$ |

1.2 When calculating we usually deal with **real numbers**. The set \mathbb{R} of real numbers can be thought of as an infinitely long continuous number line. Each point of the number line represents a real number x :



The following number sets are subsets of \mathbb{R} :

\mathbb{N} = Set of **natural numbers** = $\{1, 2, 3, 4, \dots\}$

\mathbb{Z} = Set of **integers** = $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

\mathbb{Q} = Set of **rational numbers** = $\left\{ x \mid x = \frac{m}{n}; m \in \mathbb{Z}; n \in \mathbb{Z} \setminus \{0\} \right\}$

A rational number x is a number that can be expressed as a fraction $\frac{m}{n}$, where both numerator m and denominator n are integers and n is not zero.

- a) Determine the following sets:
- | | | |
|--------------------------------------|----------------------------------|--|
| i) $\mathbb{Z} \setminus \mathbb{N}$ | ii) $\mathbb{Z} \cap \mathbb{N}$ | iii) $\mathbb{Z} \setminus \mathbb{N}$ |
|--------------------------------------|----------------------------------|--|
- b) Decide whether each statement is true or false:
- | | | |
|------------------------------------|-------------------------------------|--------------------------------------|
| i) $\mathbb{N} \subset \mathbb{Z}$ | ii) $\mathbb{Z} \subset \mathbb{Q}$ | iii) $\mathbb{Q} \subset \mathbb{R}$ |
|------------------------------------|-------------------------------------|--------------------------------------|
- c) Try to think of numbers that are elements of the set $\mathbb{R} \setminus \mathbb{Q}$

1.3 Harshbarger/Reynolds: Exercises 0.3, pages 6 to 9 (photocopies)

(Harshbarger, R.J. and Reynolds, J.J.: Mathematical Applications for the Management, Life, and Social Sciences, Houghton Mifflin Company, Boston / New York 2007, 8th edition, ISBN 978-0-618-73162-6)

Answers

- 1.1 a) $B \cap C = \{\text{Lisbon, Copenhagen, Barcelona, Naples, Stockholm, ...}\}$
b) $B \setminus C = \{\text{London, Paris, Madrid, Berlin, Rome, ...}\}$
c) $C \setminus B = \{\text{Tokyo, San Francisco, Sydney, Rio de Janeiro, ...}\}$
d) $A \setminus (B \cap C) = \{\text{Chicago, Mexico City, Nairobi, Beijing, ...}\}$

- 1.2 a) i) $\mathbb{Z} \setminus \mathbb{N} = \{0, -1, -2, -3, \dots\}$
ii) $\mathbb{Z} \cap \mathbb{N} = \mathbb{Z}$
iii) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$

- b) i) true
ii) true
iii) true

- c) $\mathbb{R} \setminus \mathbb{Q} = \{\sqrt{2}, \sqrt{3}, \dots\}$

- 1.3 see Harshbarger/Reynolds (photocopies)