

Exercise 12 Exponential function and equations Ordinary annuity, annuity due

Objectives

- be able to calculate the present and the future value of an annuity if constant payments are made at the beginning or at the end of each compounding period.
- be able to treat specific annuity tasks.

Problems

Ordinary annuity

1. Find the future value of an annuity of \$1300 paid at the end of each year for 5 years, if interest is earned at a rate of 6%, compounded annually.
2. A family wants to have a \$200'000 college fund for their children at the end of 20 years. What contribution must be made at the end of each quarter if their investment pays 7.6%, compounded quarterly?
3. If \$2500 is deposited at the end of each quarter in an account that earns 5% compounded quarterly, after how many quarters will the account contain \$80'000?
4. Assume that money on a savings account pays 1.5%, compounded annually. In order to have 20'000 CHF at the end of 10 years, ...
 - a) ... what payment must be made at the end of each year?
 - b) ... what amount has to be paid in at the beginning of the ten years if no more payments are made for the rest of the time?
5. Two twins are 23 years old and have different investment strategies.

Suppose that twin 1 invests \$2000 at the end of each year for 10 years only (until age 33) in an account that earns 8%, compounded annually. Suppose that twin 2 waits until turning 40 to begin investing.

How much must twin 2 put aside at the end of each year for the next 25 years in account that earns 8%, compounded annually, in order to have the same amount as twin 1 at the end of these 25 years (when they turn 65)?
6. Find the present value of an annuity of \$6000 paid at the end of each 6-month period for 8 years if the interest rate is 8%, compounded semiannually.
7. With a present value of \$135'000, what is the size of the withdrawals that can be made at the end of each quarter for the next 10 years if money is worth 6.4%, compounded quarterly?
8. A personal account earmarked as a retirement supplement contains \$242'000. Suppose \$200'000 is used to establish an annuity that earns 6%, compounded quarterly, and pays \$4500 at the end of each quarter. How long will it be until the account balance is \$0?

9. Mr. B owns a caravan. He could sell it now for 20'000 CHF. Alternatively, he could rent it out for 10 years at 2100 CHF per year, the rent being paid at the end of each year. After 10 years the caravan would be completely depreciated. Mr. B could invest the revenues (for either selling or renting out the caravan) at 7%, compounded annually.
Which alternative is more beneficial for Mr. B?
10. In a building loan contract 3600 CHF are paid in at the end of each year. The money earns interest at an annual rate of 3%. After 10 years twice the saved money is paid out. The debts, worth 5%, compounded annually, have to be paid off within 10 years by instalments due at the end of each year.
What is the size of the annual payments (in order to pay off the debts)?

Annuity due

11. Find the future value of an annuity due of \$100 each quarter for 2.5 years at 12%, compounded quarterly.
12. How much must be deposited at the beginning of each year in an account that pays 8%, compounded annually, so that the account will contain \$24'000 at the end of 5 years?
13. If an account that earns 5%, compounded quarterly, contains \$80'000 at the beginning and \$2500 is withdrawn at the beginning of each quarter, after how many quarters will the account contain \$0?
14. What amount must be set aside now to generate payments of \$50'000 at the beginning of each year for the next 12 years if money is worth 5.92%, compounded annually?
15. A year-end bonus of \$25'000 will generate how much money at the beginning of each month for the next year, if it can be invested at 6.48%, compounded monthly?

Miscellaneous problems

16. A couple has determined that they need \$300'000 to establish an annuity when they retire in 25 years. How much money should they deposit at the end of each month in an investment plan that pays 10%, compounded monthly, so they will have the \$300'000 in 25 years?
17. Mr. Gordon plans to invest \$300 at the end of each month in an account that pays 9%, compounded monthly. After how many months will the account be worth \$50'000?
18. Grandparents plan to open an account on their grandchild's birthday and contribute each month until she goes to college. How much must they contribute at the beginning of each month in an investment that pays 12%, compounded monthly, if they want the balance to be \$180'000 at the end of 18 years?
19. An insurance settlement of \$750'000 must replace Trixie Eden's income for the next 40 years. What income will this settlement provide at the end of each month if it is invested in an annuity that earns 8.4%, compounded monthly?
20. Juanito Domingo's parents want to establish a college trust for her. They want to make 16 quarterly withdrawals of \$2000, with the first withdrawal 3 months from now. If money is worth 7.2%, compounded quarterly, how much must be deposited now to provide for this trust?

Answers

1. $A_5 = \$7328.22$
2. $p = \$1083.40$
3. $n = 27.08\dots$ 28 quarters = 7 years
4. a) $p = 1868.70$ CHF (rounded up)
b) $C_0 = 17'233.35$ CHF (rounded up)
5. Twin 1: Capital at the age of 33 = $\$28'973.12$
Capital at the age of 65 = $\$340'059.97$
Twin 2: Capital at the age of 65 = $\$340'059.97$ (as twin 1)
Annual payment (from age 40 to age 65) = $\$4651.61$
6. $A_0 = \$69'913.77$
7. $p = \$4595.46$
8. $n = 73.78\dots$ 73 quarters (less than $\$4500$ at the end of the 74th quarter)
9. Alternative 1 (selling the caravan)
 $C_0 = 20'000$ CHF
 $C_{10} = 39'343$ CHF (rounded)
Alternative 2 (renting out the caravan)
 $A_{10} = 29'015$ CHF (rounded)
 $C_{10} > A_{10}$
alternative 1 is more beneficial
10. First 10 years (saving money)
 $A_{10} = 41'270$ CHF (rounded)
Second 10 years (paying off debts)
 $A_0 = 82'540$ CHF (rounded)
 $p = 10'690$ CHF (rounded)
11. $A_{10} = \$1180.78$
12. $p = \$3787.92$
13. $n = 40.46\dots$ 40 quarters (less than $\$2500$ at the beginning of the 41st quarter)
14. $A_0 = \$445'962.23$

15. $p = \$2145.59$

16. $p = \$266.10$

17. $n = 108.9$ (rounded) 109 months = 9 years 1 month

18. $p = \$235.16$

19. $p = \$5441.23$

20. $A_0 = \$27'590.62$