Exercise 10 Exponential function and equations
Exponential equations, logarithm, compound interest

Objectives

- be able to determine simple logarithms without a calculator.
- be able to solve simple exponential equations without a calculator.
- be able to calculate a common logarithm, a natural logarithm with a calculator.
- be able to apply one of the logarithmic properties in order to solve simple exponential equations.
- be able to treat specific compound interest tasks by means of logarithms.

Problems

1. Solve the exponential equations below **without** a calculator, i.e. find the solutions by guessing.

   a) $2^x = 16$ b) $4^x = 64$ c) $5^x = 1$

   d) $\left(\frac{3}{2}\right)^x = \frac{27}{8}$ e) $10^x = 1'000'000$ f) $10^x = 10$

2. Determine the following logarithms **without** a calculator.

   a) $\log_3(27)$ b) $\log_4(16)$ c) $\log_2(128)$

   d) $\log_{10}(1000)$ e) $\log_{10}(1)$

3. Determine the logarithms below **with** your calculator.

   a) log(1.1) b) ln(1.1) c) log(9)

   d) ln(9) e) log(2345.67) f) ln(2345.67)

4. Solve the following exponential equations.

   a) $10^x = 21$ b) $10^x = 256.78$ c) $10^x = 1'234'567$

5. Solve the exponential equations below.

   a) $3^x = 99$ b) $1.01^x = 1.5$ c) $3^{x+4} = 5$

6. An initial capital C_0 is invested at an interest rate r, compounded annually. After n years the capital amounts to C_n. Determine n.

   a) C_0 = 1000 CHF r = 1.00% C_n = 1220 CHF (rounded)

   b) C_0 = 100'000 CHF r = 2.25% C_n = 243'519 CHF (rounded)

7. How long would 10'000 CHF have to be invested at 2.5%, compounded annually, to amount to 12'000 CHF?

8. How long would any initial capital have to be invested at 1.25%, compounded annually, to double its value?

9. An initial capital of 10'000.00 CHF is invested at an unknown interest rate, compounded annually. After 10 years the capital amounts to 11'894.40 CHF. After how many years (from the beginning of the investement) will the capital be worth 15'000.00 CHF?

Answers

1. a) x = 4 b) x = 3 c) x = 0
   d) x = 3 e) x = 6 f) x = 1

2. a) 3 b) 2 c) 7
   d) 3 e) 0

3. a) 0.041... b) 0.095... c) 0.954...
   d) 2.197... e) 3.370... f) 7.760...

4. a) x = log(21) = 1.322...
   b) x = log(256.78) = 2.409...
   c) x = log(1'234'567) = 6.091...

5. a) x = 4.182... b) x = 40.748... c) x = - 2.535...

6. $n = \frac{\log\left(\frac{C_n}{C_0}\right)}{\log(q)}$
   a) n = 20
   b) n = 40

7. $n = \frac{\log\left(\frac{C_n}{C_0}\right)}{\log(q)} = \frac{\log\left(\frac{12'000}{10'000}\right)}{\log(1.025)} = 7.38...$ → 8 years

8. $C_n = C_0 \cdot q^n$
   $C_n = 2 \cdot C_0$

   $n = \frac{\log(2)}{\log(1.0125)} = 55.79...$ → 56 years

9. r = 1.75%
   C_n = 14'000 CHF for n = 23.37... → 24 years