Exercise 9 Exponential function and equations
Compound interest, exponential function

Objectives

- be able to calculate the future capital that is invested at an interest rate which is compounded annualy.
- be able to treat compound interest tasks by means of an exponential function.
- be able to determine the formula of an exponential function out of the coordinates of two points of the graph.

Problems

1. Compound interest at an annual rate r is paid on an initial capital C_0.

   a) Assume an initial capital C_0 = 1000.00 CHF, and an annual interest rate r = 2%.
   Determine the capital after one, two, three, four, and five years' time.

   b) Try to develop a formula which allows to calculate the capital C_n after n years' time for any values of C_0, r, and n.

2. What is the future capital if 8000 CHF is invested for 10 years at 12% compounded annualy?

3. What present value amounts to 10'000 CHF if it is invested for 10 years at 6% compounded annualy?

4. At what interest rate, compounded annualy, would 10'000 CHF have to be invested to amount to 14'071 CHF in 7 years?

5. Ms Smith wants to invest 150'000 CHF for five years. Bank A offers an interest rate of 6.5% compounded annualy. Bank B offers to pay 200'000 CHF after five years. Which bank makes the better offer?

6. The purchase of Alaska cost the United States $ 7 million in 1869. If this money had been placed in a savings account paying 6% compounded annualy, how much money would be available from this investment in 2010?

7. Mary Stahley invested $ 2500 in a 36-month certificate of deposit (CD) that earned 8.5% annual simple interest. When the CD matured, she invested the full amount in a mutual fund that had an annual growth equivalent to 18% compounded annualy. How much was the mutual fund worth after 9 years?

8. A capital is invested for 4 years at 4% and for 3 more years at 6%, compounded annualy. Eventually, the capital amounts to 72'000 CHF.

   a) Determine the initial capital.

   b) What was the average interest rate with respect to the whole period of time?

9. An unknown initial capital is invested at an unknown interest rate, compounded annualy. After 2 years, the capital amounts to 5'891.74 CHF, and after another 5 years the capital is 6997.54 CHF.
   Determine both initial capital and interest rate.

10. The graph of an exponential function contains the points P and Q.
    Determine the formula of the exponential function.

    a) P(0|1.02) Q(1|1.0302) b) P(1|12) Q(3|192)

Answers

1. a) $C_0 = 1000.00$ CHF
   $C_1 = 1020.00$ CHF
   $C_2 = 1040.40$ CHF
   $C_3 = 1061.21$ CHF
   $C_4 = 1082.43$ CHF
   $C_5 = 1104.08$ CHF

   b) $C_n = C_0 (1 + r)^n$

2. C(10) = 24'846.79 CHF

3. $C_0 = 5'583.95$ CHF

4. r = 5%

5. Bank A: C(5) = 205'513.00 CHF
   Bank B: C(5) = 200'000.00 CHF

6. C(141) = $ 25'896 million (rounded)

7. $ 13'916.24

8. a) $C_0 = 51'675$ CHF

   b) r = 4.85%

9. r = 3.5%
   $C_0 = 5'500.00$ CHF

10. a) $y = f(x) = 1.02 \cdot 1.01^x$

    b) $y = f(x) = 3 \cdot 4^x$