

Exercise 8 Quadratic function and equations

Quadratic function/equations, supply, demand, market equilibrium

Objectives

- be able to solve special quadratic equations without applying the quadratic formula.
- be able to solve a quadratic equation by applying the quadratic formula.
- be able to solve a quadratic equation containing a parameter.
- be able to determine the vertex form of a quadratic function out of the coordinates of the vertex and the coordinates of another point of the corresponding parabola.
- be able to determine the general form of a quadratic function out of the coordinates of three points of the corresponding parabola.
- be able to treat applied tasks in economics by means of quadratic equations or systems of quadratic equations.

Problems

- Solve the quadratic equations below without using the quadratic formula.
State the solution set for each equation.
 - $(x + 2)(x + 5) = 0$
 - $x^2 - 3x = 0$
 - $4x^2 - 9 = 0$
 - $(3x - 2)(4x + 1) = 0$
 - $3x^2 = 27$
 - $(x - 8)(5x - 9) = 0$
 - $x^2 + 7x = 0$
 - $100x^2 - 1 = 0$
 - $4x^2 + 5x = 0$
 - $x^2 = x$
- Solve the quadratic equations below. State the solution set for each equation.
 - $(7 + x)(7 - x) = (3x + 2)^2 - (2x + 3)^2$
 - $\frac{8}{x^2 - 4} + \frac{2}{2 - x} = 3x - 1$
 - $\frac{x^2 - x - 2}{2 - x} = 1$
 - $(x - 3)(2x - 7) = 1$
 - $\frac{x - 4}{x - 5} = \frac{30 - x^2}{x^2 - 5x}$
 - $\frac{x^2 - 4}{x^2 - 4} = 1$
- Determine the value(s) of the parameter b such that the quadratic equation has exactly one solution.
State this solution:
 - $2x^2 = 3x - b$
 - $x^2 + bx + b = -3$
- Solve the following equations for x. Take into account that the parameter b can have any real value.
 - $x^2 + x + b = 0$
 - $-bx = 1 + 4x^2$
- A parabola has the vertex V and contains the point P.
Determine the formula of the corresponding quadratic function both in the vertex and in the general form.
 - V(2|4) P(-1|7)
 - V(1|-8) P(2|-7)

6. A parabola contains the three points P, Q, and R.
Determine the formula of the corresponding quadratic function in the general form.
- a) P(-4|8) Q(0|0) R(10|15)
- b) P(1|-1) Q(2|4) R(4|8)
7. Find the equilibrium quantity and equilibrium price of a commodity for the given supply and demand functions f_s and f_d :
- a) supply $p = f_s(q) = \frac{1}{4}q^2 + 10$
demand $p = f_d(q) = 86 - 6q - 3q^2$
- b) supply $p = f_s(q) = q^2 + 8q + 16$
demand $p = f_d(q) = -3q^2 + 6q + 436$
8. The total costs and the total revenues for a company are given by
- $$C(x) = 2000 + 40x + x^2$$
- $$R(x) = 130x$$
- Find the break-even points.
9. The costs $C(x)$ for producing x items and the revenue $R(x)$ for selling x items are given below.
How many items are to be produced and sold in order to achieve a profit of 200 CHF?
- $$C(x) = (x^2 + 100x + 80) \text{ CHF}$$
- $$R(x) = (160x - 2x^2) \text{ CHF}$$

