Exercise 6 Linear function and equations
Linear systems of equations

Objectives

- be able to solve a linear system of equations.
- be able to treat applied tasks by means of linear systems of equations.

Problems

1. Solve the following systems of equations:

a) $4x + 3y = 14$
$2x - y = 12$

b) $- 4a - b = 40$
$a + 5b = 9$

c) $12x + 9y = 15$
$4x + 3y = 5$

d) $a - 4b = 3$
$- 5a + 20b = 10$

e) $2p - 6q = 6$
$5p + 3q = 42$

f) $2x + 3y + 5 = 5x + 6y - 1$
$x - 4y - 2 = 2x - 2y$

g) $3(x + 5) = 2(2y - 1)$
$4(3x - 6) = 3(y + 4)$

h) $(x + 5)(y + 1) = (x + 8)(y - 3)$
$(x - 3)(y - 1) = (x - 1)(y + 3)$

i) $2(2a + 3b) = 3(3a - b) + 5$
$4(3a - 4b) = 2(a + b) - 10$

2. Find the formula of the linear function whose graph contains the two points P and Q:

a) P(5|-3) Q(-2|1)

b) P(2|-3) Q(-1|-4)

c) P(3|-7) Q(3|-9)

3. Find the intersection point of the graphs of the two linear functions f and g:

a) $f: \mathbb{R} \rightarrow \mathbb{R}$, $x \rightarrow y = f(x) = - 3x + \frac{5}{4}$ $g: \mathbb{R} \rightarrow \mathbb{R}$, $x \rightarrow y = g(x) = - x - 1$

b) $f: \mathbb{R} \rightarrow \mathbb{R}$, $x \rightarrow y = f(x) = 2x + \frac{5}{4}$ $g: \mathbb{R} \rightarrow \mathbb{R}$, $x \rightarrow y = g(x) = 2x - 1$

4. Find out whether the graphs of the linear functions f, g, and h have a point P in common.

a) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = x + 1$; $g: \mathbb{R} \to \mathbb{R}$, $x \to y = g(x) = -\frac{x}{2} - 2$; $h: \mathbb{R} \to \mathbb{R}$, $x \to y = h(x) = \frac{5}{3}x + \frac{7}{3}$

b) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = \frac{1}{6}x + \frac{3}{2}$; $g: \mathbb{R} \to \mathbb{R}$, $x \to y = g(x) = -\frac{2}{3}x + 2$; $h: \mathbb{R} \to \mathbb{R}$, $x \to y = h(x) = 2x - 3$

5. Hotelier A says to hotelier B: "If three quarters of your hotel guests stayed at my hotel, I would host 100 guests." Hotelier B replies: "No, if half of your guests stayed at my hotel, I would host 100 guests."

How many guests do A and B host in their hotels?

6. The (non-linear) equation $ax^2 + bx = 1$ has the solution set S = {2, 3}, i.e. the equation has the two solutions $x_1 = 2$ and $x_2 = 3$.

Determine the values of the parameters a and b.

7. Red Tide and Blue Flake are planning new lines of skis.

Red Tide
For the first year, the fixed costs for setting up production are $45'000. The variable costs for producing each pair of skis are estimated at $80, and the selling price will be $255 per pair. It is projected that 3000 pairs will sell the first year.

Blue Flake
For the first year, the fixed costs for setting up production are $40'000. The variable costs for producing each pair of skis are estimated at $80, and the selling price will be $250 per pair. It is projected that 3500 pairs will sell the first year.

How many pairs of skis must both Red Tide and Blue Flake sell in order to realise the same profit? What is the profit?

Answers

1. a) $(x, y) = (5, -2)$

   b) $(a, b) = (-11, 4)$

   c) infinitely many solutions
   $S = \{(x, (5-4x)/3) \mid x \in \mathbb{R}\}$

   d) no solution
   $S = \{ \}$

   e) $(p, q) = (15/2, 3/2)$

   f) $(x, y) = (6, -4)$

   g) $(x, y) = (5, 8)$

   h) $(x, y) = (-2, 7)$

   i) infinitely many solutions
   $S = \{(a, 5(1+a)/9) \mid a \in \mathbb{R}\}$

2. a) $y = f(x) = -\frac{4}{7}x - \frac{1}{7}$

   b) $y = f(x) = \frac{1}{3}x - \frac{11}{3}$

   c) slope is not defined, therefore no function

3. a) $P(9/8 \mid -17/8)$

   b) no intersection point as graphs are parallel

4. a) $P(-2 \mid -1)$

   b) no intersection point P
   graphs of f and g intersect at $P(3/5 \mid 8/5)$, however graph of h does not contain P

5. A: 40 guests B: 80 guests

6. $a = -\frac{1}{6}$ $b = \frac{5}{6}$

7. Red Tide
   Total costs $C_1(x) = 80x + 45'000$
   Revenue $R_1(x) = 255x$
   Profit $P_1(x) = R_1(x) - C_1(x) = 175x - 45'000$

   Blue Flake
   Total costs $C_2(x) = 80x + 40'000$
   Revenue $R_2(x) = 250x$
   Profit $P_2(x) = R_2(x) - C_2(x) = 170x - 40'000$

   $P_2(x) = P_1(x)$

   $x = 1000$, $P_1(1000) = P_2(1000) = 130'000$
   1000 pairs of skis, profit = \$130'000