

## Exercise 5                      Linear function and equations     Linear equations

### Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to apply linear equations to exemplary tasks in economics.

### Problems

1. Determine the solution sets of the following equations:

- a)  $19x - 32 + 17x = 18x - 30 + 16x - 4$
- b)  $25x - 16 - 9x = 20 + 24x - 10 - 10x$
- c)  $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$
- d)  $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

2. Determine the solution sets of the following equations:

- a)  $22(x - 11) - 5(x - 40) = 110 - (x + 53)$
- b)  $184 - 6(x - 24) = 214 - 3(2x - 38)$
- c)  $(x + 3)(x - 5) = (x - 3)^2$
- d)  $(x - 5)(x - 2) = (x - 4)(x - 3)$
- e)  $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

3. Determine the solution sets of the following equations:

- a)  $\frac{x + 3}{5} = \frac{2x - 8}{3}$
- b)  $\frac{x + 3}{4} + \frac{1 - 3x}{7} = 0$
- c)  $\frac{2}{x - 1} = \frac{1}{x - 2}$
- d)  $\frac{x}{x - 1} = \frac{x - 1}{x - 2}$

4. Solve the following equations for  $x$ , and determine the solution sets.  
Take into account that the parameters  $p$ ,  $a$ , and  $b$  can be any real numbers.

- a)  $x(p - 3) = p$
- b)  $(a - b)x = a$

5. The graph of a linear function  $f$  with slope  $a$  contains the point  $P$ . Find the formula of the linear function.
- a)  $a = -5$                        $P(5|-3)$
  - b)  $a = 2$                                $P(3|0)$
  - c)  $a = 0$                                $P(2|3)$
6. Alps Bikes uses the formula  $B(t) = -400t + 5000$  to find the book value  $B(t)$ , in Swiss francs, of a mountain bike  $t$  years after its purchase.
- a) What do the numbers  $-400$  and  $5000$  signify?
  - b) How long will it take the mountain bike to depreciate completely?
7. Two items A and B depreciate linearly:
- |         |  |
|---------|--|
| Item A. | original value = 200 CHF<br>depreciation = 16 CHF/year |
| Item B  | original value = 240 CHF<br>depreciation = 32 CHF/year |
- a) How long will it take the two items to depreciate completely?
  - b) Determine the point in time where both items have the same value.
8. Simple interest at an unknown rate is paid on an initial bank balance of 5000 CHF. The balance after five years is 5625 CHF.
- a) Determine the interest rate.
  - b) How long would it take the balance to reach 7000 CHF?

**Answers**

1.
  - a)  $S = \{-1\}$
  - b)  $S = \{13\}$
  - c)  $S = \mathbb{R}$
  - d)  $S = \{ \}$
  
2.
  - a)  $S = \left\{ \frac{11}{2} \right\}$
  - b)  $S = \mathbb{R}$
  - c)  $S = \{6\}$
  - d)  $S = \{ \}$
  - e)  $S = \{0\}$
  
3.
  - a)  $S = \{7\}$
  - b)  $S = \{5\}$
  - c)  $S = \{3\}$
  - d)  $S = \{ \}$
  
4.
 

4.	a)	if $p = 3$ :	no solution	$S = \{ \}$
		if $p \neq 3$ :	$x = \frac{p}{p-3}$	$S = \left\{ \frac{p}{p-3} \right\}$
	b)	if $a = b = 0$ :	$x \in \mathbb{R}$	$S = \mathbb{R}$
		if $a = b \neq 0$ :	no solution	$S = \{ \}$
		if $a \neq b$ :	$x = \frac{a}{a-b}$	$S = \left\{ \frac{a}{a-b} \right\}$
  
5.
  - a)  $y = f(x) = -5x - 13$
  - b)  $y = f(x) = 2x - 6$
  - c)  $y = f(x) = 3$
  
6.
  - a) The number - 400 indicates that the value of the mountain bike decreases by 400 CHF per year. The number 5000 indicates that the original value of the mountain bike was 5000 CHF.
  - b) 12.5 years
  
7.
  - a) item A: 12.5 years  
 item B: 7.5 years
  - b)  $t = 2.5$  years
  
8.
  - a) slope of the linear function.  $a = 125$   
 interest rate = 2.5%
  - b) 16 years