Exercise 3 Function
Domain, codomain, range, graph

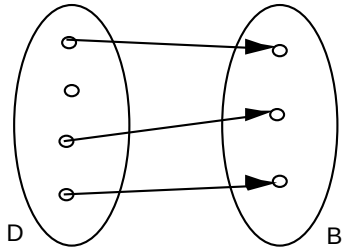
Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

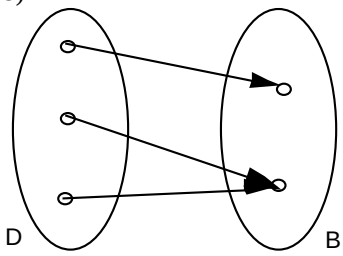
Problems

1. Which of the following relations are functions? Explain your answer.

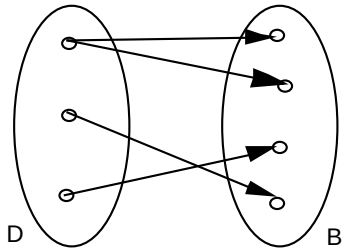
a)



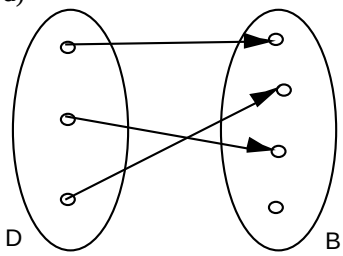
b)



c)



d)



e) D = set of all the modules of the HTW Tourism bachelor programme
B = set of all the HTW lecturers
f: D → B, m → *l* = f(m) = lecturer of m

f) D = {1978, 1979, ... , 1987, 1988}
B = set of all the human beings aged between 20 and 30
f: D → B, y → p = f(y) = person who was born in the year y

g) D = set of all the human beings aged between 20 and 30
B = {1978, 1979, ... , 1987, 1988}
f: D → B, p → y = f(p) = year of birth of person p

h) f: $\mathbb{R} \to \mathbb{R}$, x → y = f(x) = x^2

i) f: $\mathbb{R}^+ \to \mathbb{R}$, x → y = f(x) = number the square of which is x

j) f: $\mathbb{R} \to \mathbb{R}$, t → b = f(t) = bank account balance at time t

2. Determine the range E of the functions below:

   a) D = {January, February, March, ..., December}
   B = {A, B, C, ..., Z}
   f: D → B, m → *l* = f(m) = initial letter of m

   b) D = set of all the neighbouring countries of Switzerland
   B = set of all the European cities
   c: D → B, x → y = c(x) = capital of neighbouring country s

   c) function f in problem 1 g)

   d) function f in problem 1 h)

3. a) $f: \mathbb{R} \to \mathbb{R},\ x \to f(x) = x^3 - x$

   Determine the following values:

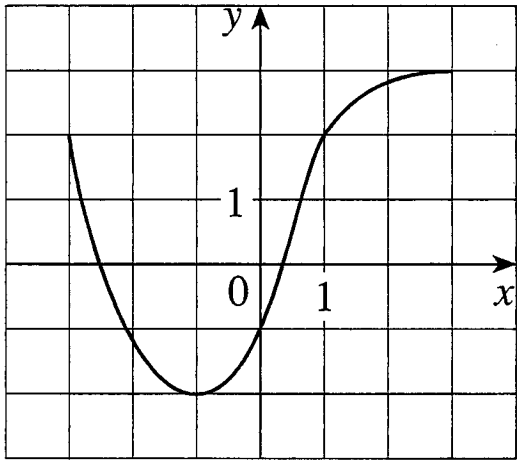
   i) f(0)
   ii) f(1)
   iv) f(a)
   v) f(x+a)

   b) $g: \mathbb{R}\setminus\{-1\} \to \mathbb{R},\ x \to g(x) = \frac{x^2}{x+1}$

   Determine the following values:

   i) g(0)
   ii) g(1)
   iv) g(a)
   v) g(x+a)

4. The graph of a function f ist given as follows:



   a) State the value of f(-1).

   b) Estimate the value of f(2).

   c) For what values of x is f(x) = 2?

   d) Estimate the values of x such that f(x) = 0.

   e) State the domain D of f.

   f) State the range E of f.

Answers

1. a) no function
   b) function
   c) no function
   d) function
   e) no function
   f) no function
   g) function
   h) function
   i) no function
   j) function

2. a) $E = \{A, D, F, J, M, N, O, S\}$
   b) $E = \{Berlin, Wien, Vaduz, Rom, Paris\}$
   c) $E = B$
   d) $E = R_0^+$

3. a) i) $f(0) = 0^3-0 = 0$
      ii) $f(1) = 1^3-1 = 0$
      iv) $f(a) = a^3-a$
      v) $f(x+a) = (x+a)^3-(x+a)$
   b) i) $g(0) = \frac{0^2}{0+1} = 0$
      ii) $g(1) = \frac{1^2}{1+1} = \frac{1}{2}$
      iv) $g(a) = \frac{a^2}{a+1}$
      v) $g(x+a) = \frac{(x+a)^2}{x+a+1}$

4. a) $f(-1) = -2$
   b) $f(2) \approx 2.8$
   c) $x_1 = -3, x_2 = 1$
   d) $x_1 \approx -2.5, x_2 \approx 0.3$
   e) $D = \{x \in R \mid -3 \leq x \leq 3\} = [-3,3]$
   f) $E = \{y \in R \mid -2 \leq x \leq 3\} = [-2,3]$