Exercise 1   Sets
Intersection, complement, union, number sets

Objectives

- understand what a set, an element of a set, an intersection, a union, a complement is.
- know the set of real numbers, rational numbers, integers, natural numbers.
- be able to perform basic set operations.

Problems

1.   Look at the sets A, B, and C:

   A = Set of all the cities of the world
   B = Set of all the European cities
   C = Set of all the coastal cities of the world

   Find at least four elements of the following sets:

   a)   B ∩ C

   b)   B \ C

   c)   C \ B

   d)   A \ (B ∩ C)

2.   When calculating we usually deal with **real numbers**. The set $\mathbb{R}$ of real numbers can be thought of as an infinitely long continuous number line. Each point of the number line represents a real number x:
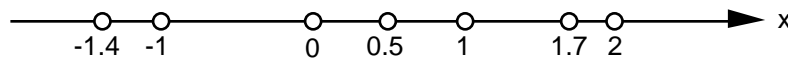
   The following number sets are subsets of $\mathbb{R}$:

   $\mathbb{N}$ = Set of **natural numbers** = {1, 2, 3, 4, ...}

   $\mathbb{Z}$ = Set of **integers** = {..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}

   $\mathbb{Q}$ = Set of **rational numbers** = $\left\{ x \mid x = \dfrac{m}{n} ; m \in \mathbb{Z}; n \in \mathbb{Z}; n \neq 0 \right\}$

   A rational number x is a number that can be expressed as a fraction $\dfrac{m}{n}$, where both numerator m and denominator n are integers and n is not zero.

   a)   Determine the following sets:

      i) $\mathbb{Z} \setminus \mathbb{N}$

      ii) $\mathbb{Z} \cup \mathbb{N}$

      iii) $\mathbb{Z} \cap \mathbb{N}$

   b)   Decide whether each statement is true or false:

      i) $\mathbb{N} \subseteq \mathbb{Z}$

      ii) $\mathbb{Z} \subseteq \mathbb{Q}$

      iii) $\mathbb{Q} \subseteq \mathbb{R}$

   c)   Try to think of numbers that are elements of the set $\mathbb{R} \setminus \mathbb{Q}$

Answers

1.   a) $B \cup C$ = {Lisbon, Copenhagen, Barcelona, Naples, Stockholm, ...}

        b) $B \setminus C$ = {London, Paris, Madrid, Berlin, Rome, ...}

        c) $C \setminus B$ = {Tokyo, San Francisco, Sydney, Rio de Janeiro, ...}

        d) $A \setminus (B \cup C)$ = {Chicago, Mexico City, Nairobi, Beijing, ...}

2.   a)   i) $\mathbb{Z} \setminus \mathbb{N}$ = {0, -1, -2, -3, ...}

             ii) $\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$

             iii) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$

        b)   i)   true

             ii)   true

             iii)   true

        c) $\mathbb{R} \setminus \mathbb{Q}$ = {$\sqrt{2}, \sqrt{3}, \quad , ...$}