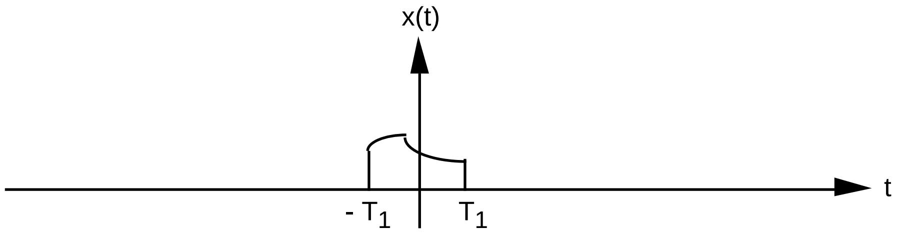
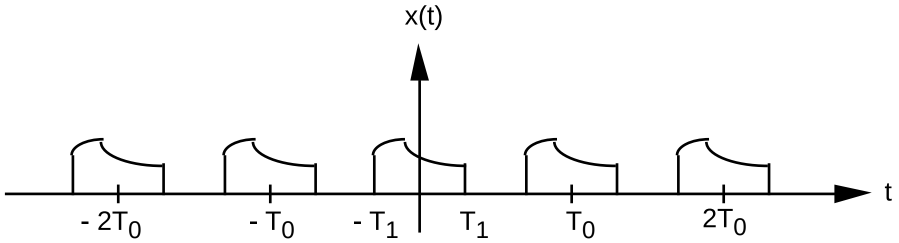
Fourier-Transformierte und Fourier-Integral

Fourier-Integral als Grenzwert einer Fourier-Reihe

Aperiodische Funktion bzw. aperiodisches Signal x(t) von endlicher Dauer:





$$x(t) = \lim_{T_0 \to \infty} \tilde{x}(t)$$

Fourier-Reihe von $\tilde{x}(t)$:

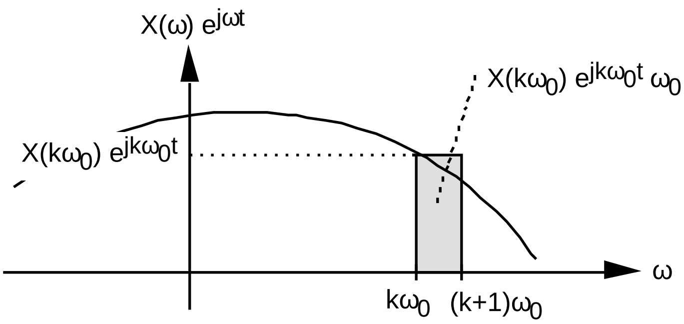
$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{jk\omega_0 t}$$

$$\begin{aligned}
c_k &= \frac{1}{T_0} \int_{(T_0)} \tilde{x}(t) \cdot e^{-jk\omega_0 t}\, dt \\
&= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \tilde{x}(t) \cdot e^{-jk\omega_0 t}\, dt \\
&= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cdot e^{-jk\omega_0 t}\, dt \\
&= \frac{1}{T_0} \int_{-\infty}^{\infty} x(t) \cdot e^{-jk\omega_0 t}\, dt
\end{aligned}$$

Def.: $X(\omega) := \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$

$$= \frac{1}{T_0} X(k\omega_0)$$

$$\begin{aligned}\tilde{x}(t) &= \sum_{k=-\infty}^{\infty} \frac{1}{T_0} X(k\omega_0)\, e^{jk\omega_0 t} \\ &= \sum_{k=-\infty}^{\infty} \frac{\omega_0}{2\pi} X(k\omega_0)\, e^{jk\omega_0 t} \\ &= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(k\omega_0)\, e^{jk\omega_0 t}\, \omega_0\end{aligned}$$



Fourier-Reihe von $\tilde{x}(t)$: $\tilde{x}(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(k\omega_0)\, e^{jk\omega_0 t}\, \omega_0$

$$\Big\downarrow \quad T_0 \to \infty \text{ bzw. } \omega_0 \to 0$$

Fourier-Integral von x(t): $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$

Fourier-Transformierte und Fourier-Integral

Aperiodische Funktion bzw. aperiodisches Signal x(t):

$$\boxed{\begin{aligned} X(\omega) &:= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \\ x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega \end{aligned}}$$

Fourier-Transformierte von x(t)

Fourier-Integral von x(t)