# Komplexe Zahlen

### Menge der komplexen Zahlen

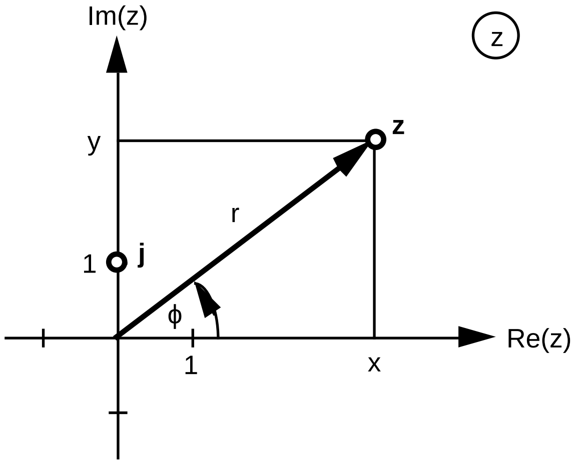
C := {z | z = x + j·y   x   R    y   R}

j = imaginäre Einheit
$j^2 = -1$

### Darstellung einer komplexen Zahl

| | |
|---|---|
| Kartesische Form | $z = x + j\cdot y$ |
| Polarform | $z = r\cdot\big(\cos(\phi) + j\cdot\sin(\phi)\big) = r\cdot \mathrm{cis}(\phi)$ |
| Exponentialform | $z = r\cdot e^{j\phi}$ |

Gauss'sche Zahlenebene




| | |
|---|---|
| $x = \mathrm{Re}(z)$ | Realteil |
| $y = \mathrm{Im}(z)$ | Imaginärteil |
| $r = \lvert z \rvert$ | Betrag |
| $\phi = \arg(z)$ | Argument |

### Euler'sche Formel

$e^{j\phi} = \cos(\phi) + j\cdot\sin(\phi)$

Bew.: Potenzreihenentwicklung

$$\cos(\phi) = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + - \ldots$$

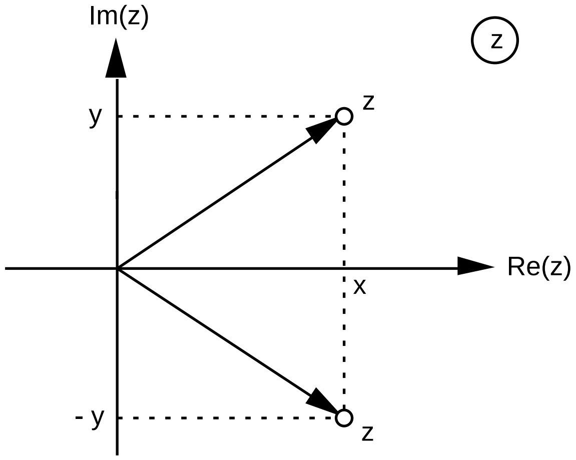
$$\sin(\phi) = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + - \ldots$$

$$\begin{aligned}
e^{j\phi} &= 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \frac{(j\phi)^5}{5!} + \frac{(j\phi)^6}{6!} + \ldots \\
&= 1 + j\phi - \frac{\phi^2}{2!} - j\,\frac{\phi^3}{3!} + \frac{\phi^4}{4!} + j\,\frac{\phi^5}{5!} - \frac{\phi^6}{6!} - \ldots \\
&= \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - + \ldots\right) + j\left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - + \ldots\right) \\
&= \cos(\phi) + j\cdot\sin(\phi)
\end{aligned}$$

**Komplexe Konjugation**

$z = x + j \cdot y$

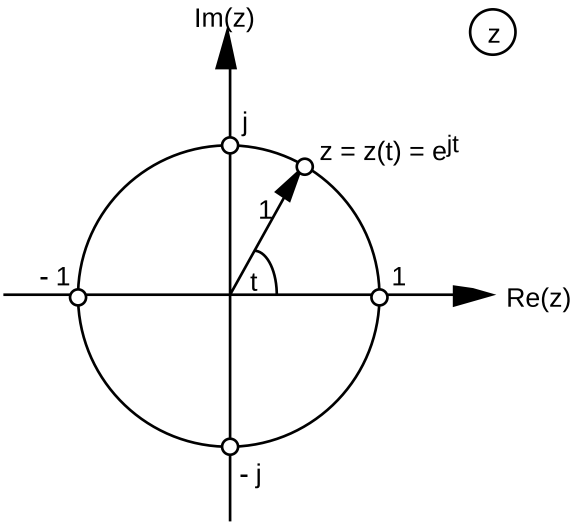
$z^* := x - j \cdot y$ komplex konjugierte Zahl



# Komplexe Exponentialfunktion

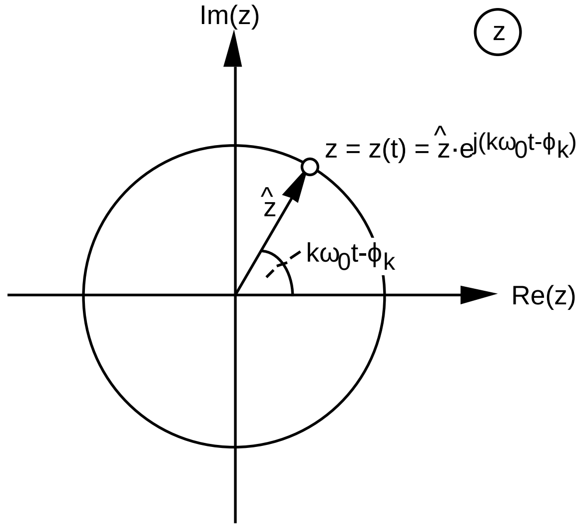
$z: \quad \mathbb{R} \to \mathbb{C}$

$\quad t \to z = z(t) = e^{jt}$

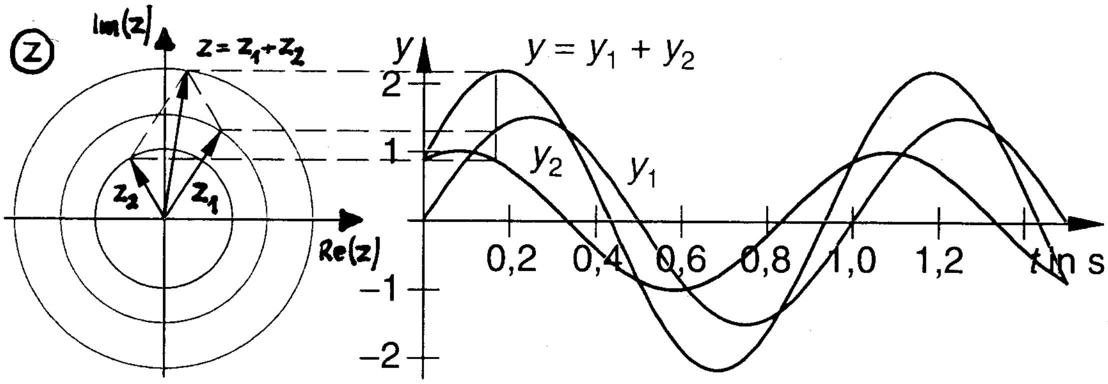



$z: \quad \mathbb{R} \to \mathbb{C}$

$\quad t \to z = z(t) = \hat{z} \cdot e^{j(k\omega_0 t - \phi_k)}$




**Zeigerdiagramm**




$y_1(t) = \hat{y}_1 \sin(\omega_0 t)$

$z_1(t) = \hat{y}_1 e^{j\omega_0 t}$

$y_1(t) = \mathrm{Im}(z_1(t)) = \mathrm{Im}\left(\hat{y}_1 e^{j\omega_0 t}\right)$

$y_2(t) = \hat{y}_2 \sin(\omega_0 t - \phi)$

$z_2(t) = \hat{y}_2 e^{j(\omega_0 t - \phi)}$

$y_2(t) = \mathrm{Im}(z_2(t)) = \mathrm{Im}\left(\hat{y}_2 e^{j(\omega_0 t - \phi)}\right)$