Komplexe Zahlen

Menge der komplexen Zahlen

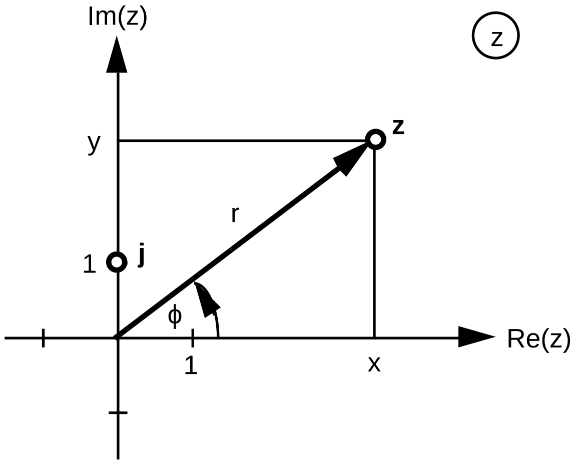
C := {z | z = x + j·y   x R   y R}

j = imaginäre Einheit
$j^2 = -1$

Darstellung einer komplexen Zahl

Kartesische Form	$z = x + j\cdot y$
Polarform	$z = r\cdot\big(\cos(\phi) + j\cdot\sin(\phi)\big) = r\cdot\mathrm{cis}(\phi)$
Exponentialform	$z = r\cdot e^{j\phi}$

Gauss'sche Zahlenebene



$x = \mathrm{Re}(z)$	Realteil
$y = \mathrm{Im}(z)$	Imaginärteil
$r = \|z\|$	Betrag
$\phi = \arg(z)$	Argument

Euler'sche Formel

$e^{j\phi} = \cos(\phi) + j\cdot\sin(\phi)$

Bew.: Potenzreihenentwicklung

$$\cos(\phi) = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + - \ldots$$

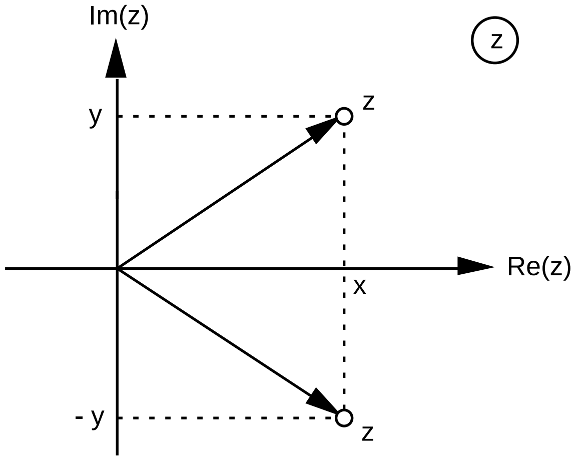
$$\sin(\phi) = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + - \ldots$$

$$\begin{aligned} e^{j\phi} &= 1 + j\phi + \frac{(j\phi)^2}{2!} + \frac{(j\phi)^3}{3!} + \frac{(j\phi)^4}{4!} + \frac{(j\phi)^5}{5!} + \frac{(j\phi)^6}{6!} + \ldots \\ &= 1 + j\phi - \frac{\phi^2}{2!} - j\,\frac{\phi^3}{3!} + \frac{\phi^4}{4!} + j\,\frac{\phi^5}{5!} - \frac{\phi^6}{6!} - \ldots \\ &= \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - + \ldots\right) + j\left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - + \ldots\right) \\ &= \cos(\phi) + j\cdot\sin(\phi) \end{aligned}$$

Komplexe Konjugation

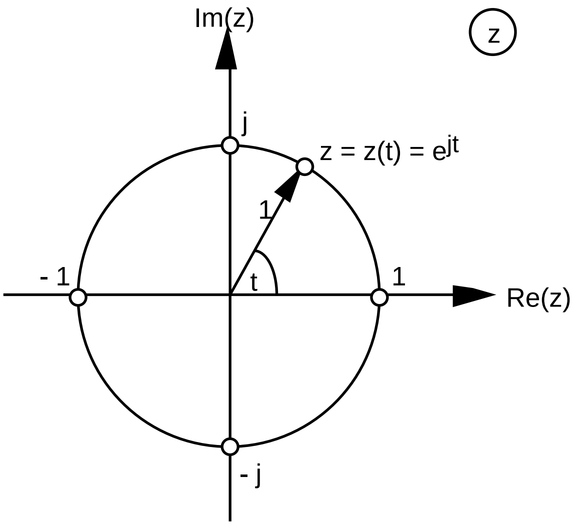
$z = x + j \cdot y$

$z^* := x - j \cdot y$ komplex konjugierte Zahl

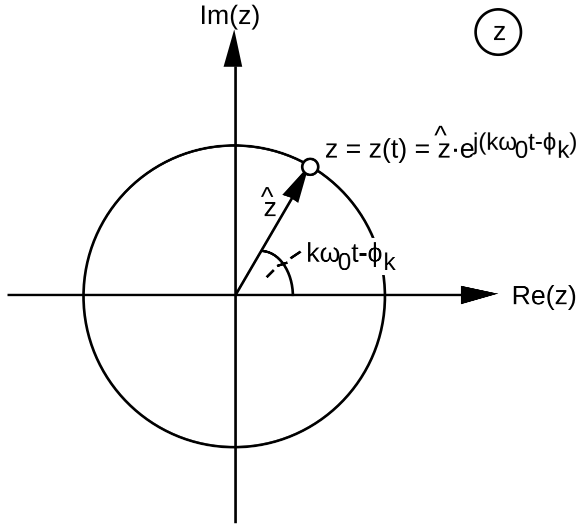


Komplexe Exponentialfunktion

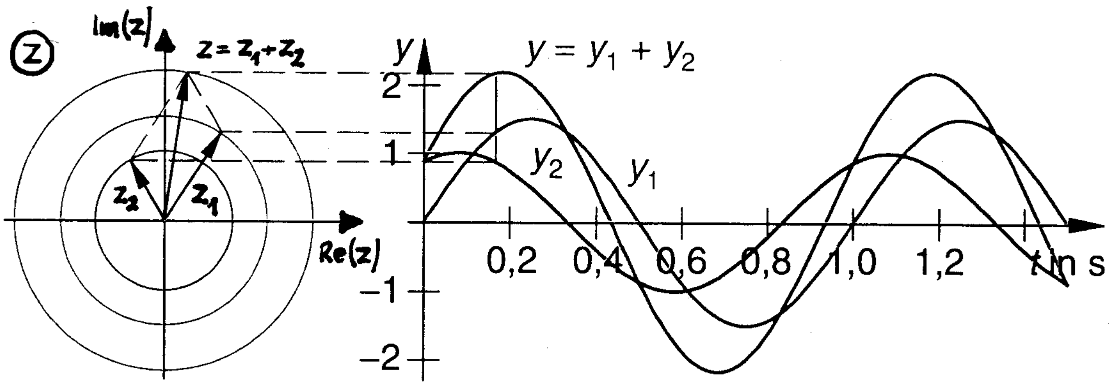
$x: \quad \mathbb{R} \to \mathbb{C}$
$\quad t \to z = z(t) = e^{jt}$



$x: \quad \mathbb{R} \to \mathbb{C}$
$\quad t \to z = z(t) = \hat{z} \cdot e^{j(k\omega_0 t - \phi_k)}$



Zeigerdiagramm



$y_1(t) = \hat{y}_1 \sin(\omega_0 t)$

$z_1(t) = \hat{y}_1 e^{j\omega_0 t}$

$y_1(t) = \mathrm{Im}(z_1(t)) = \mathrm{Im}\left(\hat{y}_1 e^{j\omega_0 t}\right)$

$y_2(t) = \hat{y}_2 \sin(\omega_0 t - \phi)$

$z_2(t) = \hat{y}_2 e^{j(\omega_0 t - \phi)}$

$y_2(t) = \mathrm{Im}(z_2(t)) = \mathrm{Im}\left(\hat{y}_2 e^{j(\omega_0 t - \phi)}\right)$