Review exercises 2 Differential calculus, integral calculus

Problems

R2.1 Decide whether the statements below are true or false:

a) "The derivative (derived function) of a function is a function."

b) "The derivative (rate of change) of a function at a particular position is a number."

c) "The function f has a local maximum at $x = x_1$ if $f'(x_1) = 0$ and $f''(x_1) > 0$."

d) "If $f''(x_2) = 0$ and $f'''(x_2) < 0$, then the function f has a point of inflection at $x = x_2$."

e) "If $g' = f$, then g is an antiderivative of f."

f) "f with $f(x) = 2x + 20$ is an antiderivative of g with $g(x) = x^2$."

g) "f with $f(x) = 3x$ has infinitely many antiderivatives."

h) "The indefinite integral of a function is a set of functions."

R2.2 Determine the value $f(x_0)$, the first derivative $f'(x_0)$, and the second derivative $f''(x_0)$ of the function f at the position x_0:

a) $f(x) = 4x^2(x^2 - 1)$ $x_0 = -1$

b) $f(x) = (-3x^2 + 2x - 1)\cdot e^x$ $x_0 = -2$

c) $f(x) = (x^2 + 2)\cdot e^{-3x}$ $x_0 = -\frac{1}{3}$

R2.3 For the given cost function C(x) and revenue function R(x) determine ...

i) ... the marginal cost function C'(x).

ii) ... the marginal revenue function R'(x).

iii) ... the marginal profit function P'(x).

a) $C(x) = (40x + 200)$ CHF $R(x) = 60x$ CHF

b) $C(x) = (5x^2 + 20x + 100)$ CHF $R(x) = (-2x^2 + 100x)$ CHF

c) $C(x) = (20x^2 + 50 + 3e^{4x})$ CHF $R(x) = \left(200x - e^{-4x^2}\right)$ CHF

R2.4 Determine all positions where the given function has …

i) … a local maximum.
… a local minimum.

ii) … a point of inflection.

a) $f(x) = 2x^3 - 9x^2 + 12x - 1$

b) f(x) as in R2.2 a)

R2.5 The total revenue function for a commodity or a service is given by

$$R(x) = (-0.01x^2 + 36x) \text{ CHF}$$

The production is limited to at most 1500 units. Determine the maximum revenue.

R2.6 The total cost function for a commodity or a service is given by

$$C(x) = (x^2 + 100) \text{ CHF}$$

Determine the number of units x that will result in a minimum average cost.

Determine that minimum average cost, too.

R2.7 A firm can produce 1000 units per month only. The monthly total cost is given by

$$C(x) = (200x + 300) \text{ CHF}$$

where x is the number of items produced. The total revenue is given by

$$R(x) = \left(-\frac{1}{100}x^2 + 250x\right) \text{ CHF}$$

Determine the number of items that will result in a maximum profit.

Determine that maximum profit, too.

R2.8 Determine the indefinite integrals below:

a) $\int (x^4 - 3x^3 - 6)\ dx$

b) $\int \left(\frac{1}{2}x^6 - \frac{2}{3x^4}\right)\ dx$

R2.9 The equation of the third derivative f ''' of a function f is given as follows:

$$f'''(x) = 3x + 1$$

Determine the equation of the function f such that $f''(0) = 0$, $f'(0) = 1$, and $f(0) = 2$.

R2.10 The marginal cost for producing a product or rendering a service is $C'(x) = (5x + 10)$ CHF, with a fixed cost of 800 CHF.

Determine the cost of producing or rendering 20 units.

R2.11 A certain firm's marginal cost $C'(x)$ and the derivative of the average revenue $\overline{R}'(x)$ are given as follows:

$$C'(x) = (6x + 60) \text{ CHF}$$

$$\overline{R}'(x) = -1 \text{ CHF}$$

If 10 items are produced or rendered, the total costs are 1000 CHF, and the revenue is 1700 CHF.

Determine the number of units x that will result in a maximum profit.

Determine that maximum profit, too.

R2.12 The supply function for a product or service is

$$p = f_s(x) = (4x + 4) \text{ CHF}$$

and the demand function is

$$p = f_d(x) = (-x^2 + 49) \text{ CHF}$$

Determine the equilibrium point and both the consumer's and the producer's surplus.

R2.13 (see next page)

R2.13 The supply function for a product or a service is

$$p = f_s(x) = \left(ax^2 - \frac{6}{5}x + 2\right) \text{ CHF}$$

and the demand function is

$$p = f_d(x) = (-\,bx^2 + 110) \text{ CHF}$$

with unknown parameters a and b. The equilibrium price is 10 CHF, and the producer's surplus is 73.33 CHF (rounded).

Determine the two unknown parameters a and b.

Hint:
- Use the unrounded value $\left(73 + \frac{1}{3}\right)$ CHF = $\frac{220}{3}$ CHF for the producer's surplus.