

Review exercises 1 Functions and equations

Problems

R1.1 Decide which of the following relations are functions. Give reasons for your answers.

- a) $f_1 : \mathbb{R}_0^+ \rightarrow \mathbb{R}^+, x \mapsto y = f_1(x) = \sqrt{x}$
- b) $f_2 : \{2, 3, 4, \dots\} \rightarrow \mathbb{N}, x \mapsto y = f_2(x) = x - 1$
- c) $D = \text{Set of all Swiss cantons}$
 $C = \text{Set of all Swiss towns and cities}$
 $f_3 : D \rightarrow C, x \mapsto y = f_3(x) = \text{capital of } x$
- d) $f_4 : \{x : x \in \mathbb{R} \text{ and } x \geq 3\} \rightarrow \mathbb{R}, x \mapsto y = f_4(x) = \frac{1}{x^2 - 9}$
- e) $f_5 : \mathbb{R}_0^+ \rightarrow \mathbb{R}, x \mapsto y = f_5(x) = \log_a(x)$

R1.2 If $f(x) = 9x - x^2$, find ...

- a) ... $f(0)$.
- b) ... $f(-3)$.
- c) ... $\frac{f(x+h) - f(x)}{h}$ and simplify the expression.

R1.3 Solve the equations below, and state the solution sets:

- a) $3x - 8 = 23$
- b) $\frac{6}{3x-5} = \frac{6}{2x+3}$
- c) $\frac{2x+5}{x+7} = \frac{1}{3} + \frac{x-11}{2x+14}$

R1.4 Solve the following equations for x , and state the solution sets.
Take into account that the parameters a and p can be any real numbers.

- a) $ax = 60$
- b) $(p - 1)px = p^2 - 1$

R1.5 Solve each system of equations, and state the solution sets:

- a) $2x + y = 19$
 $x - 2y = 12$
- b) $6x + 3y = 1$
 $y = -2x + 1$

R1.6 Determine the equation of the linear function whose graph ...

- a) ... has slope 4 and intercept 2.
- b) ... passes through $(-2|1)$ and has slope $\frac{2}{5}$.
- c) ... (see next page)

- c) ... passes through $(-2|7)$ and $(6|-4)$.
d) ... passes through $(1|6)$ and is parallel to $y = 4x - 6$.

R1.7 A certain product has the following supply and demand functions:

$$p = f_s(q) = (4q + 5) \text{ CHF}$$

$$p = f_d(q) = (-2q + 81) \text{ CHF}$$

- a) Determine both the number of units supplied and the number of units demanded if the price is 53 CHF.
b) Determine both the equilibrium quantity and the equilibrium price.

R1.8 The total cost and total revenue for a certain product are given by the following:

$$C(x) = (38.80x + 4500) \text{ CHF}$$

$$R(x) = 61.30x \text{ CHF}$$

- a) Determine the fixed costs.
b) Determine the variable costs for producing 10 units.
c) Determine the number of units required to break even.

R1.9 The supply and the demand functions for a product are linear and determined by the tables below:

Supply function		Demand function	
Price	Quantity	Price	Quantity
100 CHF	200	200 CHF	200
200 CHF	400	100 CHF	400
300 CHF	600	0 CHF	600

Determine the quantity and price that will give market equilibrium.

R1.10 Determine the solution sets of the equations below:

- a) $4x - 3x^2 = 0$
b) $3x^2 - 6x = 9$
c) $4x^2 + 25 = 0$
d) $\frac{1}{x} + 2x = \frac{1}{3} + \frac{x+1}{x}$
e) $\frac{x-4}{x-5} = \frac{30-x^2}{x^2-5x}$

R1.11 Determine the equation of the quadratic function whose graph ...

- a) ... has the vertex $(2|4)$ and passes through $(3|3)$.
b) ... passes through $(-3|-3)$, $(0|3)$, and $(3|0)$.

R1.12 The supply function for a product is given by $p = (q^2 + 300)$ CHF, and the demand is given by $p/\text{CHF} + q = 410$.

Determine the equilibrium quantity and price.

R1.13 The total costs for a service (x = service quantity) are given by $C(x) = (1760 + 8x + 0.6x^2)$ CHF and the total revenues are given by $R(x) = (100x - 0.4x^2)$ CHF.

Determine the break-even points.

R1.14 Determine the equation of the exponential function whose graph passes through P and Q.

a) $P(0|1)$ $Q(2|9)$

b) $P(1|20)$ $Q(2|100)$

R1.15 Evaluate each logarithm without using a calculator:

a) $\log_5(1)$

b) $\log_2(8)$

c) $\log_3\left(\frac{1}{3}\right)$

d) $\log_3(3^8)$

e) $e^{\ln(5)}$

f) $10^{\lg(3.15)}$

R1.16 8000 CHF are borrowed for 3 years at an annual simple interest rate of 12%.

Determine the future value of the loan at the end of the 3 years.

R1.17 Mary borrowed 2000 CHF from her parents and repaid them 2200 CHF after 4 years.

Determine the annual simple interest rate at which Mary borrowed the money.

R1.18 A future student wants to deposit money on a bank account in order to have 3000 CHF for tuition and fees 3 years later. The investment pays simple interest at an annual interest rate of 6%.

Determine the amount of money the future student must pay in.

R1.19 Assume that 1000 CHF are invested for 4 years at a nominal annual interest rate of 8%, compounded quarterly.

Determine the amount of interest that will be earned in the 4 years.

R1.20 An investment pays interest at a nominal annual interest rate of 5.4%, compounded monthly. The goal is to have 18'000 CHF in 4 years.

Determine the amount of money that has to be invested.

R1.21 In 2010 an African country had a population of 4.5 million. The population has been increasing at 4% per year.

Determine what the population will be in 2030 if the growth factor does not change.

R1.22 (see next page)

- R1.22 A company wants to have 250'000 CHF available in 4 1/2 years for new construction. The investment pays interest at a nominal annual interest rate of 10.2%, compounded quarterly.
- Determine how much must be deposited at the beginning of each quarter to reach the mentioned goal.
- R1.23 A retirement account that pays interest at a nominal annual interest rate of 6.8%, compounded semiannually, contains 488'000 CHF.
- Determine how long 40'000 CHF can be withdrawn at the end of each half-year until the account balance is 0 CHF.
- R1.24 Three years from now, a couple plan to spend 4 months travelling in China, Japan, and Southeast Asia. When they take their trip, they would like to withdraw 5000 CHF at the beginning of each month to cover their expenses for that month. Assume that the account pays interest at a nominal annual interest rate of 6.6%, compounded monthly.
- Determine how much the couple must deposit at the beginning of each month for the next 3 years so that there is enough money in their account when they start travelling.
- R1.25 Mr S. is obligated to pay 25'000 CHF at the end of each of the following 8 years to his divorced wife. As a result of a personal profit in his company, he is able to pay the whole sum at the end of the first year (instead of making 8 payments at the end of each year). Assume that interest, compounded annually, has been fixed at an annual interest rate of 4.5%.
- Determine the amount of money Mr. S. has to pay at the end of the first year.
- R1.26 Mr P. is thinking about an investment for his retirement. He would like to withdraw 8000 CHF from an account at the end of each year for 15 years starting at the end of the year in which he turns 60. He assumes that money pays interest at an annual interest rate of 2.5% throughout the 15 years.
- a) He wants to save the money by making 30 constant payments at the end of each year until turning 55. He assumes that his bank pays him interest at an annual interest rate of 3%, compounded annually, and that the interest rate of 3% is valid until he turns 60.
- Determine how much Mr P. must pay in each year.
- b) Mr P. has won 40'000 CHF in a lottery! Assume that he pays the money in at the end of the year in which he turns 25 and that there are the same interest conditions as in a).
- Decide whether this amount is sufficient for his retirement scheme. Give reasons for your answer.