

Exercises 14 Indefinite integral Antiderivative, indefinite integral, coefficient/sum rule

Objectives

- be able to determine an antiderivative and the indefinite integral of a constant, basic power, and basic exponential function.
- be able to apply the coefficient and sum rules to determine the indefinite integral of a function.
- be able to determine the cost, revenue, and profit functions if the marginal cost, marginal revenue, and marginal profit functions are known.

Problems

14.1 Determine the indefinite integrals below:

- | | |
|----------------------------|----------------------------|
| a) $\int x^2 dx$ | b) $\int x^3 dx$ |
| c) $\int x^{-5} dx$ | d) $\int \frac{1}{x^2} dx$ |
| e) $\int \frac{1}{x^4} dx$ | f) $\int 4 dx$ |
| g) $\int (-7) dx$ | h) $\int e^x dx$ |
| i) $\int e^{3x} dx$ | j) $\int e^{-x} dx$ |

14.2 Determine the indefinite integral of the following functions f:

- | | |
|--|--|
| a) $f(x) = x^5$ | b) $f(x) = 3x^2$ |
| c) $f(x) = x^3 + 2x^2 - 5$ | d) $f(x) = \frac{x^5}{2} - \frac{2}{3x^2}$ |
| e) $f(x) = \frac{1}{2}x^3 - 2x^2 + 4x - 5$ | f) $f(x) = x^{10} - \frac{1}{2}x^3 - x$ |

14.3 Determine the equations of those two antiderivatives F_1 and F_2 of f which fulfil the stated conditions.

- | | | |
|--------------------------|--------------|---------------|
| a) $f(x) = 10x^2 + x$ | $F_1(0) = 3$ | $F_2(0) = -1$ |
| b) $f(x) = x^3 + 3x + 1$ | $F_1(2) = 5$ | $F_2(4) = -8$ |

14.4 Suppose that we know the equation of the derivative f' of a function f :

$$f'(x) = 3x^2 - 50x + 250$$

Determine the equation of the function f , if ...

- | |
|-------------------------|
| a) ... $f(0) = 500$. |
| b) ... $f(10) = 2500$. |

14.5 Suppose that we know the equation of the second derivative f'' of a function f :

$$f''(x) = 2x - 1$$

Determine the equation of the function f such that $f'(2) = 4$ and $f(1) = -1$.

14.6 The monthly marginal cost for a product or a service is $C'(x) = (2x + 100)$ CHF, with fixed costs amounting to 200 CHF.

Determine the total cost function for a month.

- 14.7 The marginal cost for a product or a service is $C'(x) = (4x + 2)$ CHF, and the production or rendering of 10 units results in a total cost of 300 CHF.
Determine the total cost function.
- 14.8 The marginal cost for a product or a service is $C'(x) = (4x + 40)$ CHF, and the total cost of producing or rendering 25 units is 3000 CHF.
Determine the total cost for 30 units.
- 14.9 A firm knows that its marginal cost for a service is $C'(x) = (3x + 20)$ CHF, that its marginal revenue is $R'(x) = (-5x + 44)$ CHF, and that the cost of rendering of 10 units is 370 CHF.
Determine the ...
a) ... profit function $P(x)$.
b) ... number of units that results in a maximum profit.
Hint:
- The revenue R is zero if no unit is sold. Thus, $R(0) = 0$ CHF.
- 14.10 Suppose that the marginal revenue $R'(x)$ and the derivative of the average cost $\bar{C}'(x)$ of a company are given as follows:
$$R'(x) = 400 \text{ CHF}$$
$$\bar{C}'(x) = \left(\frac{2}{15}x - 11 - \frac{10'000}{x^2} \right) \text{ CHF}$$

Producing or rendering 15 units results in a total cost of 16'750 CHF.
Determine the ...
a) ... profit function $P(x)$.
b) ... number of units that results in a maximum profit.
c) ... maximum profit.
- 14.11 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.
- a) An antiderivative of a function is a ...
☐ ... real number.
☐ ... function.
☐ ... set of functions.
☐ ... graph.
- b) The indefinite integral of a function is a ...
☐ ... real number.
☐ ... function.
☐ ... set of functions.
☐ ... graph.
- c) (see next page)

c) If $f = g'$ then ...

- ☐ ... f is an antiderivative of g .
- ☐ ... g is an antiderivative of f .
- ☐ ... f is the indefinite integral of g .
- ☐ ... g is the indefinite integral of f .