

## Exercises 14      Indefinite integral Antiderivative, indefinite integral, coefficient/sum rule

### Objectives

- be able to determine an antiderivative and the indefinite integral of a constant, basic power, and basic exponential function.
- be able to apply the coefficient and sum rules to determine the indefinite integral of a function.
- be able to determine the cost, revenue, and profit functions if the marginal cost, marginal revenue, and marginal profit functions are known.

### Problems

14.1 Determine the indefinite integrals below:

- |                            |                            |
|----------------------------|----------------------------|
| a) $\int x^2 dx$           | b) $\int x^3 dx$           |
| c) $\int x^{-5} dx$        | d) $\int \frac{1}{x^2} dx$ |
| e) $\int \frac{1}{x^4} dx$ | f) $\int 4 dx$             |
| g) $\int (-7) dx$          | h) $\int e^x dx$           |
| i) $\int e^{3x} dx$        | j) $\int e^{-x} dx$        |

14.2 Determine the indefinite integral of the following functions f:

- |                                            |                                            |
|--------------------------------------------|--------------------------------------------|
| a) $f(x) = x^5$                            | b) $f(x) = 3x^2$                           |
| c) $f(x) = x^3 + 2x^2 - 5$                 | d) $f(x) = \frac{x^5}{2} - \frac{2}{3x^2}$ |
| e) $f(x) = \frac{1}{2}x^3 - 2x^2 + 4x - 5$ | f) $f(x) = x^{10} - \frac{1}{2}x^3 - x$    |

14.3 Determine the equations of those two antiderivatives  $F_1$  and  $F_2$  of  $f$  which fulfil the stated conditions.

- |                          |              |               |
|--------------------------|--------------|---------------|
| a) $f(x) = 10x^2 + x$    | $F_1(0) = 3$ | $F_2(0) = -1$ |
| b) $f(x) = x^3 + 3x + 1$ | $F_1(2) = 5$ | $F_2(4) = -8$ |

14.4 Suppose that we know the equation of the derivative  $f'$  of a function  $f$ :

$$f'(x) = 3x^2 - 50x + 250$$

Determine the equation of the function  $f$ , if ...

- |                         |
|-------------------------|
| a) ... $f(0) = 500$ .   |
| b) ... $f(10) = 2500$ . |

14.5 Suppose that we know the equation of the second derivative  $f''$  of a function  $f$ :

$$f''(x) = 2x - 1$$

Determine the equation of the function  $f$  such that  $f'(2) = 4$  and  $f(1) = -1$ .

14.6 The monthly marginal cost for a product or a service is  $C'(x) = (2x + 100)$  CHF, with fixed costs amounting to 200 CHF.

Determine the total cost function for a month.

- 14.7 The marginal cost for a product or a service is  $C'(x) = (4x + 2)$  CHF, and the production or rendering of 10 units results in a total cost of 300 CHF.  
Determine the total cost function.
- 14.8 The marginal cost for a product or a service is  $C'(x) = (4x + 40)$  CHF, and the total cost of producing or rendering 25 units is 3000 CHF.  
Determine the total cost for 30 units.
- 14.9 A firm knows that its marginal cost for a service is  $C'(x) = (3x + 20)$  CHF, that its marginal revenue is  $R'(x) = (-5x + 44)$  CHF, and that the cost of rendering of 10 units is 370 CHF.  
Determine the ...  
a) ... profit function  $P(x)$ .  
b) ... number of units that results in a maximum profit.  
Hint:  
- The revenue  $R$  is zero if no unit is sold. Thus,  $R(0) = 0$  CHF.
- 14.10 Suppose that the marginal revenue  $R'(x)$  and the derivative of the average cost  $\bar{C}'(x)$  of a company are given as follows:  
$$R'(x) = 400 \text{ CHF}$$
$$\bar{C}'(x) = \left( \frac{2}{15}x - 11 - \frac{10'000}{x^2} \right) \text{ CHF}$$
  
Producing or rendering 15 units results in a total cost of 16'750 CHF.  
Determine the ...  
a) ... profit function  $P(x)$ .  
b) ... number of units that results in a maximum profit.  
c) ... maximum profit.
- 14.11 Decide which statements are true or false. Put a mark into the corresponding box.  
In each problem a) to c), exactly one statement is true.
- a) An antiderivative of a function is a ...  
☐ ... real number.  
☐ ... function.  
☐ ... set of functions.  
☐ ... graph.
- b) The indefinite integral of a function is a ...  
☐ ... real number.  
☐ ... function.  
☐ ... set of functions.  
☐ ... graph.
- c) (see next page)

c) If  $f = g'$  then ...

- ☐ ...  $f$  is an antiderivative of  $g$ .
- ☐ ...  $g$  is an antiderivative of  $f$ .
- ☐ ...  $f$  is the indefinite integral of  $g$ .
- ☐ ...  $g$  is the indefinite integral of  $f$ .

## Answers

14.1    a)     $\int x^2 dx = \frac{1}{3}x^3 + C$                       b)     $\int x^3 dx = \frac{1}{4}x^4 + C$   
           c)     $\int x^{-5} dx = -\frac{1}{4x^4} + C$                       d)     $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$   
           e)     $\int \frac{1}{x^4} dx = -\frac{1}{3x^3} + C$                       f)     $\int 4 dx = 4x + C$   
           g)     $\int (-7) dx = -7x + C$                       h)     $\int e^x dx = e^x + C$   
           i)     $\int e^{3x} dx = \frac{1}{3}e^{3x} + C$                       j)     $\int e^{-x} dx = -e^{-x} + C$

14.2    a)     $\int f(x) dx = \int x^5 dx = \frac{1}{6}x^6 + C$   
           b)     $\int f(x) dx = \int 3x^2 dx = x^3 + C$   
           c)     $\int f(x) dx = \int (x^3 + 2x^2 - 5) dx = \frac{1}{4}x^4 + \frac{2}{3}x^3 - 5x + C$   
           d)     $\int f(x) dx = \int \left(\frac{1}{2}x^5 - \frac{2}{3x^2}\right) dx = \frac{1}{12}x^6 + \frac{2}{3x} + C$   
           e)     $\int f(x) dx = \int \left(\frac{1}{2}x^3 - 2x^2 + 4x - 5\right) dx = \frac{1}{8}x^4 - \frac{2}{3}x^3 + 2x^2 - 5x + C$   
           f)     $\int f(x) dx = \int \left(x^{10} - \frac{1}{2}x^3 - x\right) dx = \frac{1}{11}x^{11} - \frac{1}{8}x^4 - \frac{1}{2}x^2 + C$

14.3    a)     $F_1(x) = \frac{10}{3}x^3 + \frac{1}{2}x^2 + 3$                        $F_2(x) = \frac{10}{3}x^3 + \frac{1}{2}x^2 - 1$   
           b)     $F_1(x) = \frac{1}{4}x^4 + \frac{3}{2}x^2 + x - 7$                        $F_2(x) = \frac{1}{4}x^4 + \frac{3}{2}x^2 + x - 100$

Hints:

- First, determine the indefinite integral of f.
- Then, determine the value of the integration constant such that the stated conditions are fulfilled.

14.4    a)     $f(x) = x^3 - 25x^2 + 250x + 500$   
           b)     $f(x) = x^3 - 25x^2 + 250x + 1500$

14.5     $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x - \frac{17}{6}$

14.6     $C(x) = (x^2 + 100x + 200)$  CHF

Hints:

- First integrate the marginal cost function  $C'(x) \Rightarrow C(x) = (x^2 + 100x + C)$  CHF ( $C \in \mathbb{R}$ )
- Determine the integration constant C using the fact that  $C(0) = 200$  CHF  $\Rightarrow C = 200$

14.7     $C(x) = (2x^2 + 2x + 80)$  CHF

14.8     $C(30) = 3750$  CHF

Hint:

- First, determine the cost function  $C(x) \Rightarrow C(x) = (2x^2 + 40x + 750)$  CHF.

14.9    (see next page)

14.9 a)  $P(x) = (-4x^2 + 24x - 20)$  CHF

Hints:

- First, determine the cost and revenue functions  $C(x)$  and  $R(x)$ .

$$\Rightarrow C(x) = \left(\frac{3}{2}x^2 + 20x + 20\right) \text{ CHF}$$

$$R(x) = \left(-\frac{5}{2}x^2 + 44x\right) \text{ CHF}$$

- Then, determine the profit function  $P(x)$ .

b) 3 units

Hints:

- The profit function  $P(x)$  is a quadratic function.

- Think of the graph of the profit function when determining the global maximum.

14.10 a)  $P(x) = \left(-\frac{1}{15}x^3 + 11x^2 - 200x - 10'000\right)$  CHF

Hints:

- First, determine the revenue function  $R(x) \Rightarrow R(x) = 400x$  CHF

- Then, determine the average cost function  $\bar{C}(x) \Rightarrow \bar{C}(x) = \left(\frac{1}{15}x^2 - 11x + \frac{10'000}{x} + C\right)$  CHF

- Then, determine the total cost function  $C(x) \Rightarrow C(x) = \left(\frac{1}{15}x^3 - 11x^2 + 600x + 10'000\right)$  CHF

- Finally, determine the profit function  $P(x) \Rightarrow P(x) = R(x) - C(x) = \dots$

b) 100 units

Hints:

- Determine the local maxima of the profit function  $P(x)$ .

- Check if one of the local maxima is the global maximum.

c)  $P_{\max} = P(100) = 13'333$  CHF (rounded)

14.11 a) 2<sup>nd</sup> statement

b) 3<sup>rd</sup> statement

c) 2<sup>nd</sup> statement