

Exercises 2

Function Domain, codomain, range, graph

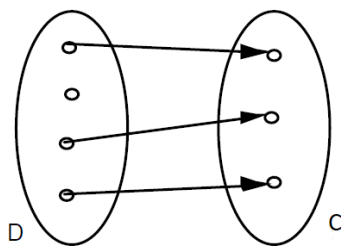
Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

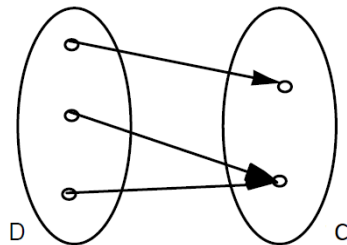
Problems

2.1 Decide which of the following relations are functions. Give reasons for your answers.

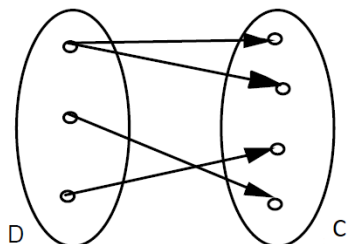
a)



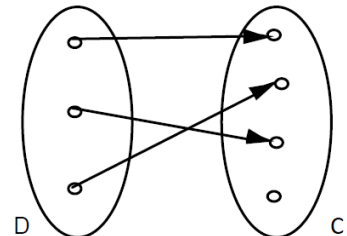
b)



c)



d)



- e) D = set of all courses in the FHGR Tourism bachelor programme
 C = set of all FHGR lecturers
 $f: D \rightarrow C, c \mapsto l = f(c)$ = lecturer of course c

- f) $D = \{1995, 1996, \dots, 2004, 2005\}$
 $C = \text{set of all human beings aged between 20 and 30}$
 $f: D \rightarrow C, y \mapsto p = f(y) = \text{person who was born in the year } y$
- g) $D = \text{set of all human beings aged between 20 and 30}$
 $C = \{1995, 1996, \dots, 2004, 2005\}$
 $f: D \rightarrow C, p \mapsto y = f(p) = \text{year of birth of person } p$
- h) $f: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto y = f(x) = x^2$
- i) $f: \mathbb{R}^+ \rightarrow \mathbb{R}, x \mapsto y = f(x) = \text{number whose square is } x$
 Notice:
 - $\mathbb{R}^+$ is the set of all positive real numbers, i.e. $\mathbb{R}^+ = \{x: x \in \mathbb{R} \text{ and } x > 0\}$.
- j) $f: \mathbb{R} \rightarrow \mathbb{R}, t \mapsto b = f(t) = \text{bank account balance at time } t$

2.2 Determine the range R of the functions below:

- a) $D = \{\text{January, February, March, ..., December}\}$
 $C = \{A, B, C, ..., Z\}$
 $f: D \rightarrow C, m \mapsto l = f(m) = \text{initial letter of month } m$
- b) $D = \text{set of all neighbouring countries of Switzerland}$
 $C = \text{set of all European towns and cities}$
 $c: D \rightarrow C, x \mapsto y = c(x) = \text{capital of neighbouring country } x$
- c) function f in problem 2.1 g)
- d) function f in problem 2.1 h)

2.3 a) $f: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto f(x) = x^3 - x$

Determine the following values:

- | | | | | | |
|-----|----------|-----|------------|------|--------------|
| i) | $f(1)$ | ii) | $f(-2)$ | iii) | $f(a)$ |
| iv) | $f(b^2)$ | v) | $f(a - b)$ | vi) | $f(x^3 - x)$ |

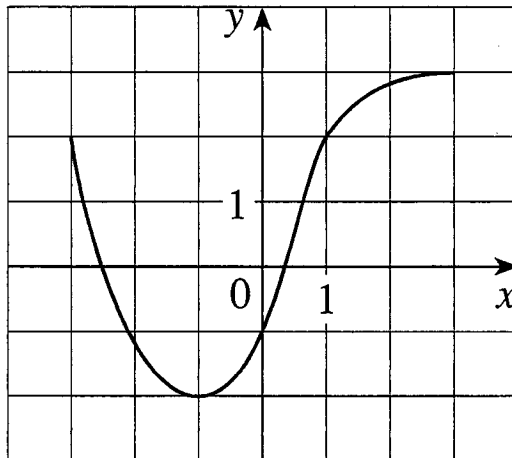
b) $g: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}, x \mapsto g(x) = \frac{x^2}{x+1}$

Determine the following values:

- | | | | | | |
|-----|----------|-----|------------|------|---------------------------------|
| i) | $g(2)$ | ii) | $g(-3)$ | iii) | $g(a)$ |
| iv) | $g(b^2)$ | v) | $g(a - b)$ | vi) | $g\left(\frac{x^2}{x+1}\right)$ |

2.4 (see next page)

2.4 The graph of a function f is given as follows:



- State the value of $f(-1)$.
- Estimate the value of $f(2)$.
- State all values of x such that $f(x) = 2$.
- Estimate all values of x such that $f(x) = 0$.
- State the domain D of f .
- State the range R of f .

2.5 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

- The range of the function $f: \{x: x \in \mathbb{R} \text{ and } x \geq 4\} \rightarrow \mathbb{R}, x \mapsto y = f(x) = \sqrt{x - 4}$, is the set ...
 - ☐ ... $\{x: x \in \mathbb{R} \text{ and } x \geq 4\}$
 - ☐ ... $\{y: y \in \mathbb{R} \text{ and } y \geq 4\}$
 - ☐ ... $\mathbb{R}$
 - ☐ ... $\mathbb{R}_0^+$
- f cannot be a function if ...
 - ☐ ... the domain of f is no number set.
 - ☐ ... the codomain of f contains more elements than the domain of f .
 - ☐ ... the domain of f contains more elements than the codomain of f .
 - ☐ ... at least one element of the domain of f has more than one image.
- If the range of a function contains as many elements as the domain, it can be concluded that ...
 - ☐ ... the range is the same set as the domain.
 - ☐ ... the codomain contains as many elements as the domain.
 - ☐ ... each element of the codomain is also an element of the range.
 - ☐ ... no element of the range is associated to more than one element of the domain.