

Exercises 1

Number sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, set operations

Objectives

- know the definition and elements of natural numbers, integers, rational numbers, and real numbers.
- know and understand what a set, an element of a set, an empty set, a subset, an intersection, a union, and a set difference are.
- be able to perform basic set operations.

Problems

1.1 Decide whether each statement is true or false:

- | | | |
|------------------------------------|--|---|
| a) $4 \in \mathbb{N}$ | b) $-\frac{14}{7} \in \mathbb{Z}$ | c) $\sqrt{2} \in \mathbb{Q}$ |
| d) $\sqrt{9} \in \mathbb{N}$ | e) $\sqrt{9} \in \mathbb{Q}$ | f) $\sqrt{9} \in \mathbb{R}$ |
| g) $1.67854 \in \mathbb{Q}$ | h) $1.\overline{67854} \in \mathbb{Q}$ | i) $\mathbb{N} \subset \mathbb{Z}$ |
| j) $\mathbb{Z} \subset \mathbb{Q}$ | k) $\mathbb{Q} \subset \mathbb{R}$ | l) $\mathbb{R} \setminus \mathbb{Z} = \mathbb{N}$ |

1.2 Determine the following sets:

- | | | |
|--|--|--|
| a) $\mathbb{Z} \setminus \mathbb{N}$ | b) $\mathbb{Z} \cup \mathbb{N}$ | c) $\mathbb{Z} \cap \mathbb{N}$ |
| d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$ | e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$ | f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N}$ |

1.3 Look at the sets A, B, and C:

A = Set of all cities of the world
B = Set of all European cities
C = Set of all coastal cities of the world

Find at least five elements of the following sets:

- | | |
|--------------------|-----------------------------|
| a) $B \cap C$ | b) $B \setminus C$ |
| c) $C \setminus B$ | d) $A \setminus (B \cup C)$ |

1.4 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

- a)
- | | |
|--------------------------|---|
| ☐ | $\mathbb{N} \cup \mathbb{Z} = \mathbb{Q}$ |
| ☐ | $\mathbb{Q} \setminus \mathbb{Z} = \mathbb{N}$ |
| ☐ | $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$ |
| ☐ | $\mathbb{Z} \setminus \mathbb{N} = \{-1, -2, -3, \dots\}$ |
- b)
- A = Set of all cities of the world
B = Set of all European cities
- | | |
|--------------------------|----------------|
| ☐ | $A \cap B = A$ |
| ☐ | $A \cup B = B$ |
| ☐ | $B \in A$ |
| ☐ | $B \subset A$ |
- c) (see next page)

- c) Assume that x is a rational number. Therefore, it can be concluded that x is ...
- ☐ ... a real number.
 - ☐ ... an integer.
 - ☐ ... a fraction where both numerator and denominator are natural numbers.
 - ☐ ... a natural number.

Answers

- 1.1 a) true b) true c) false
 d) true e) true f) true
 g) true h) true i) true
 j) true k) true l) false
- 1.2 a) $\mathbb{Z} \setminus \mathbb{N} = \{0, -1, -2, -3, \dots\}$
 b) $\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$
 c) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$
 d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q}) = \{\}$
 e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$
 f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N} = \{\}$
- 1.3 a) $B \cap C = \{\text{Lisbon, Copenhagen, Barcelona, Naples, Stockholm, ...}\}$
 b) $B \setminus C = \{\text{London, Paris, Madrid, Berlin, Rome, ...}\}$
 c) $C \setminus B = \{\text{Tokyo, San Francisco, Sydney, Rio de Janeiro, Cape Town, ...}\}$
 d) $A \setminus (B \cup C) = \{\text{Chicago, Mexico City, Nairobi, Beijing, Bogotá, ...}\}$
- 1.4 a) 3rd statement
 b) 4th statement
 c) 1st statement