

Indefinite integral

Ex.: Financial mathematics

Given the marginal cost function C' for producing a commodity or rendering a service:

$$C'(x) = (3x + 50) \text{ CHF}$$

What is the cost function C ?

$$C(x) = \dots ?$$

General problem

Given a function f . What function F is such that $F' = f$?

Ex.: 1. $f(x) = 2x$

$$\begin{aligned} \Rightarrow \quad & F_1(x) = x^2 && \text{as } F_1'(x) = 2x = f(x) \\ & F_2(x) = x^2 + 1 && \text{as } F_2'(x) = 2x + 0 = 2x = f(x) \\ & F_3(x) = x^2 - 4 && \text{as } F_3'(x) = 2x + 0 = 2x = f(x) \\ & \dots && \\ & F(x) = x^2 + C \quad (C \in \mathbb{R}) && \text{as } F'(x) = 2x + 0 = 2x = f(x) \end{aligned}$$

These are all functions F with $F' = f$. There are no additional functions F with equations different from $F(x) = x^2 + C \quad (C \in \mathbb{R})$.

2. $f(x) = 8x^3$

$$\begin{aligned} \Rightarrow \quad & F_1(x) = 2x^4 && \text{as } F_1'(x) = 8x^3 = f(x) \\ & F_2(x) = 2x^4 + 5 && \text{as } F_2'(x) = 8x^3 + 0 = 8x^3 = f(x) \\ & F_3(x) = 2x^4 - 11 && \text{as } F_3'(x) = 8x^3 + 0 = 8x^3 = f(x) \\ & \dots && \\ & F(x) = 2x^4 + C \quad (C \in \mathbb{R}) && \text{as } F'(x) = 8x^3 + 0 = 8x^3 = f(x) \end{aligned}$$

Definitions

F is called an **antiderivative** of f if its derivative F' is equal to f , i.e. $F'(x) = f(x)$.

The set of all antiderivatives of the function f is called the **indefinite integral** of f , denoted $\int f(x) \, dx$.

Ex.: 1. $f(x) = 8x^3$

All antiderivatives F have the form $F(x) = 2x^4 + C \quad (C \in \mathbb{R})$.
Therefore, we write $\int 8x^3 \, dx = \int 8x^3 \, dx = 2x^4 + C$

2. $f(x) = 12x^2$
 $\int f(x) \, dx = \int 12x^2 \, dx = 4x^3 + C$

3. $\int 2x \, dx = x^2 + C$

4. $\int 3 e^{3x} \, dx = e^{3x} + C$

$C \quad (C \in \mathbb{R})$ is called the **integration constant**.