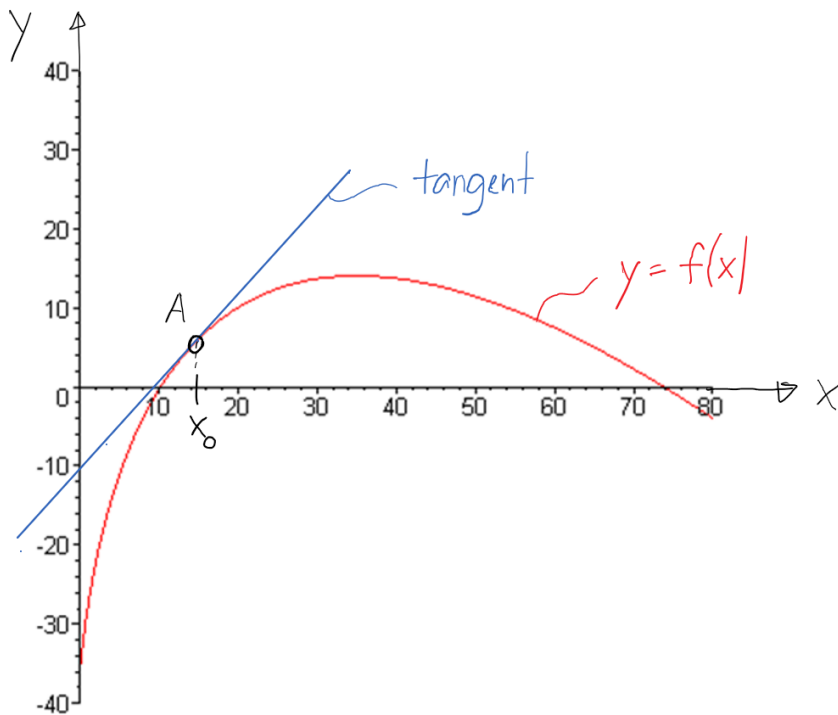


# Derivative

## Function $f$

$f: D \rightarrow \mathbb{R}$  where  $D \subseteq \mathbb{R}$   
 $x \mapsto y = f(x)$

Ex.:  $f(x) = 24\sqrt{x+1} - 2x - 60$



**What** do we want to know?

**Slope of the tangent** to the graph of the function  $f$  at a certain point  $A(x_0 | f(x_0))$ .

**Why** do we want to know the slope?

- **rate of change** of values  $y$  with respect to  $x$
- **increasing** (slope  $> 0$ ), **decreasing** (slope  $< 0$ )
- local **maximum/minimum** (slope = 0)
- **concavity** (concave up if slope increases, concave down if slope decreases), points of inflection

Applications in economics

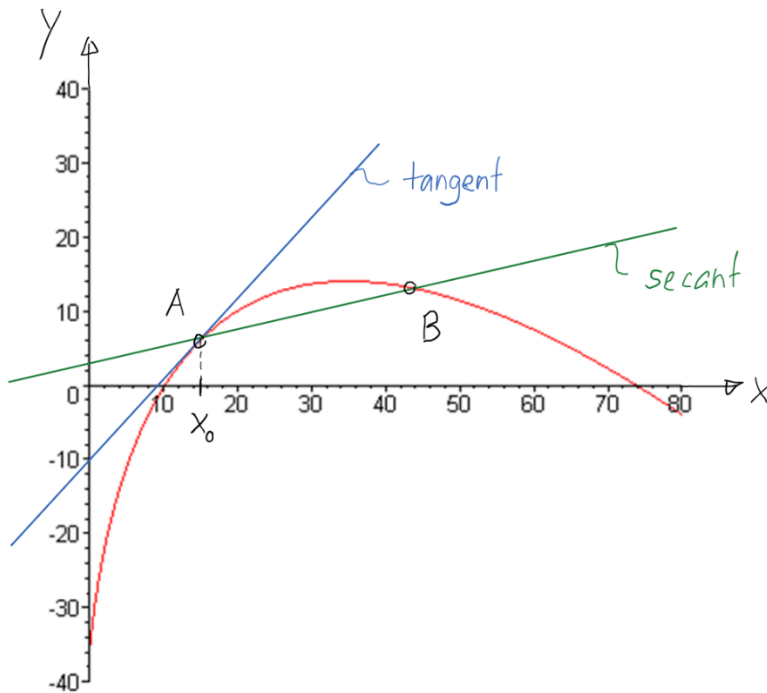
- tendency of costs/revenue/profit
- maximum/minimum of costs/revenue/profit
- **marginal costs/revenue/profit**: change of costs/revenue/profit if  $x$  (number of items or quantity of service) increases by one unit

## Definition

The slope of the tangent to the graph of  $f$  through the point  $A(x_0 | f(x_0))$  is called the **derivative** (or **rate of change**) of  $f$  at  $x_0$ , denoted  $f'(x_0)$  (“f prime of  $x_0$ ”).

**How** can we determine the slope?

The slope of the **secant** through the points  $A(x_0 | f(x_0))$  and  $B(x_0 + \Delta x | f(x_0 + \Delta x))$  tends towards the slope of the **tangent** through  $A(x_0 | f(x_0))$  as  $\Delta x$  tends towards 0.



Ex.:  $f: \mathbb{R} \rightarrow \mathbb{R}$   
 $x \mapsto y = f(x) = x^2$   
 $f'(x_0) = 2x_0$  (without proof)

Ex.:  $f: D \rightarrow \mathbb{R}$   
 $x \mapsto y = f(x) = 24\sqrt{x+1} - 2x - 60$   
 $f'(x_0) = \frac{12}{\sqrt{x_0+1}} - 2$  (without proof)

**Definition**

Suppose that the derivative (rate of change)  $f'(x_0)$  exists for all  $x_0 \in D_1$ , where  $D_1 \subseteq D$ .  
 The function  $f'$   
 $f': D_1 \rightarrow \mathbb{R}$   
 $x \mapsto y = f'(x)$   
 is called the **derivative** (or **derived function**) of  $f$ .

Ex.:  $f: \mathbb{R} \rightarrow \mathbb{R}$   
 $x \mapsto y = f(x) = x^2$

$f': \mathbb{R} \rightarrow \mathbb{R}$   
 $x \mapsto y = f'(x) = 2x$

Ex.:  $f: D \rightarrow \mathbb{R}$   
 $x \mapsto y = f(x) = 24\sqrt{x+1} - 2x - 60$

$f': D_1 \rightarrow \mathbb{R}$   
 $x \mapsto y = f'(x) = \frac{12}{\sqrt{x+1}} - 2$

