Exponential function

Definition

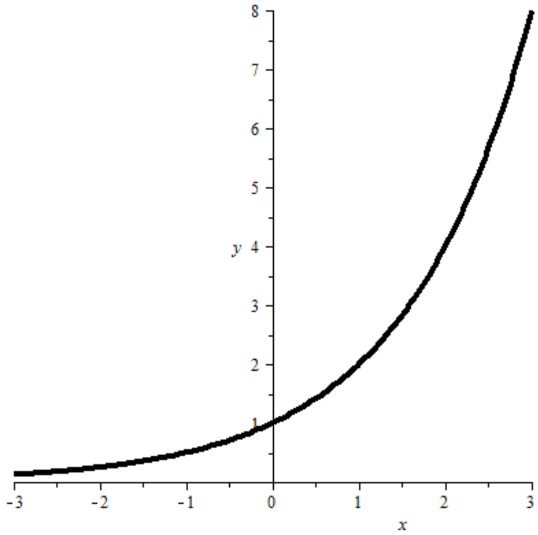
f: $D \to \mathbb{R}$ ($D \subseteq \mathbb{R}$)

$x \mapsto y = f(x) = c \cdot a^x$ ($a \in \mathbb{R}^+ \setminus \{1\}$, $c \in \mathbb{R} \setminus \{0\}$)

$a > 1$: exponential **growth**
$a < 1$: exponential **decay**

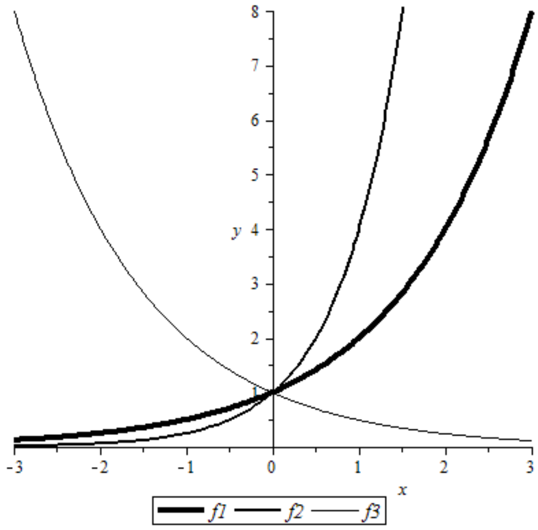
Graph

1. $y = f(x) = 2^x$ ($c = 1$, $a = 2$)



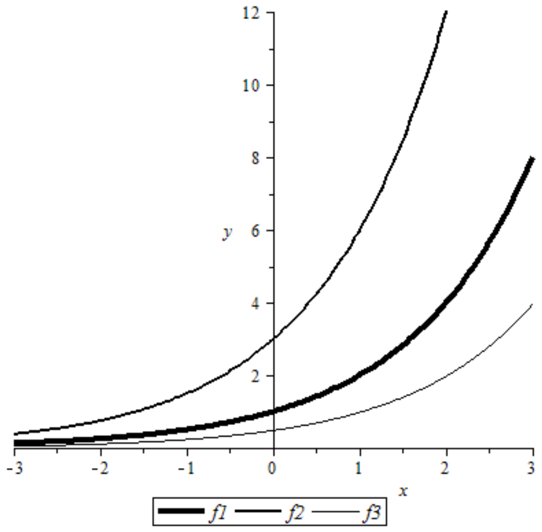
2. Parameter **a** (**a is varied**, c is kept constant)

$y = f_1(x) = 2^x$ ($c = 1$, $\mathbf{a = 2}$)
$y = f_2(x) = 4^x$ ($c = 1$, $\mathbf{a = 4}$)
$y = f_3(x) = \left(\frac{1}{2}\right)^x$ $\left(c = 1, \mathbf{a = \frac{1}{2}}\right)$

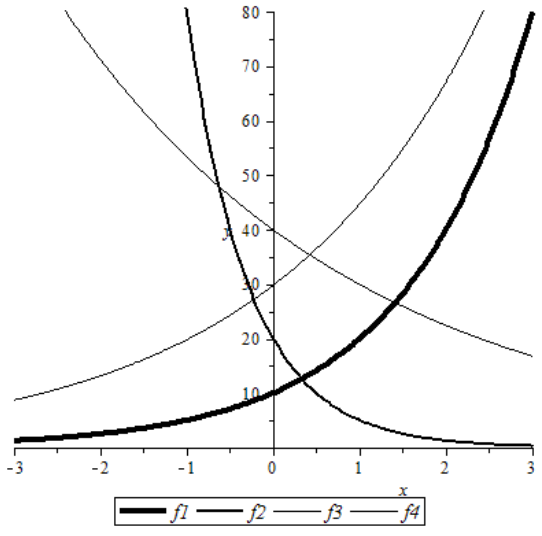


3. Parameter **c** (**c is varied**, a is kept constant)

$y = f_1(x) = 2^x$ (**c = 1**, a = 2)
$y = f_2(x) = 3 \cdot 2^x$ (**c = 3**, a = 2)
$y = f_3(x) = \frac{1}{2} \cdot 2^x$ $\left(\mathbf{c = \frac{1}{2}}, a = 2\right)$



4. $y = f_1(x) = 10 \cdot 2^x$ (c = 10, a = 2)
$y = f_2(x) = 20 \cdot 0.25^x$ (c = 20, a = 0.25)
$y = f_3(x) = 40 \cdot 0.75^x$ (c = 40, a = 0.75)
$y = f_4(x) = 30 \cdot 1.5^x$ (c = 30, a = 1.5)



Examples

1. Compound interest (exponential **growth**)

$C_n = C_0 \cdot q^n$

C_0 = initial capital
C_n = capital after n compounding periods
n = number of compounding periods (often: 1 compounding period = 1 year)
q = interest/growth factor = 1 + r (r > 0, q > 1)
r = interest rate per compounding period

Ex.: $C_0 := 1000$ CHF, $r := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow C_n = 1000 \cdot 1.02^n$ CHF

2. Depreciation (exponential **decay**)

$P(t) = P_0 \cdot q^t$

P_0 = initial price / initial purchasing power
P(t) = price / purchasing power at time t (often: t = number of years)
q = decay factor = 1 + r (r < 0, q < 1)

Ex.: $P_0 := 100$ CHF, $r := -3\% = -0.03 \Rightarrow q = 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t$ CHF