Review exercises 2 — Differential calculus, integral calculus

Problems

R2.1 Decide whether the statements below are true or false:

a) "The derivative (derived function) of a function is a function."

b) "The derivative (rate of change) of a function at a particular position is a number."

c) "The function f has a local maximum at $x = x_1$ if $f'(x_1) = 0$ and $f''(x_1) > 0$."

d) "If $f''(x_2) = 0$ and $f'''(x_2) < 0$, then the function f has a point of inflection at $x = x_2$."

e) "If $g' = f$, then g is an antiderivative of f."

f) "f with $f(x) = 2x + 20$ is an antiderivative of g with $g(x) = x^2$."

g) "f with $f(x) = 3x$ has infinitely many antiderivatives."

h) "The indefinite integral of a function is a set of functions."

R2.2 Determine the value $f(x_0)$, the first derivative $f'(x_0)$, and the second derivative $f''(x_0)$ of the function f at the position x_0:

a) $f(x) = 4x^2(x^2 - 1)$ $\quad x_0 = -1$

b) $f(x) = (-3x^2 + 2x - 1)\cdot e^x$ $\quad x_0 = -2$

c) $f(x) = (x^2 + 2)\cdot e^{-3x}$ $\quad x_0 = -\frac{1}{3}$

R2.3 For the given cost function C(x) and revenue function R(x) determine ...

i) ... the marginal cost function C'(x).

ii) ... the marginal revenue function R'(x).

iii) ... the marginal profit function P'(x).

a) $C(x) = (40x + 200)$ CHF $\quad R(x) = 60x$ CHF

b) $C(x) = (5x^2 + 20x + 100)$ CHF $\quad R(x) = (-2x^2 + 100x)$ CHF

c) $C(x) = (20x^2 + 50 + 3e^{4x})$ CHF $\quad R(x) = \left(200x - e^{-4x^2}\right)$ CHF

R2.4 For the function f, determine …

i) … the local maxima and minima.

ii) … the points of inflection.

a) $f(x) = 2x^3 - 9x^2 + 12x - 1$

b) f(x) as in R2.2 a)

R2.5 The total revenue function for a commodity or a service is given by

$$R(x) = (-0.01x^2 + 36x) \text{ CHF}$$

Determine the maximum revenue if production is limited to at most 1500 units.

R2.6 If the total cost function for a commodity or a service is

$$C(x) = (x^2 + 100) \text{ CHF}$$

producing or rendering how many units x will result in a minimum average cost?
Determine that minimum average cost.

R2.7 A firm can produce 1000 units per month only. The monthly total cost is given by

$$C(x) = (200x + 300) \text{ CHF}$$

where x is the number produced. The total revenue is given by

$$R(x) = \left(-\frac{1}{100}x^2 + 250x\right) \text{ CHF}$$

How many items should the firm produce for a maximum profit?
Determine that maximum profit.

R2.8 Determine the indefinite integrals below:

a) $\int (x^4 - 3x^3 - 6)\ dx$

b) $\int \left(\frac{1}{2}x^6 - \frac{2}{3x^4}\right)\ dx$

R2.9 The equation of the third derivative f''' of a function f is given as follows:

$$f'''(x) = 3x + 1$$

Determine the equation of the function f such that $f''(0) = 0$, $f'(0) = 1$, $f(0) = 2$

R2.10 The marginal cost for producing a product or rendering a service is $C'(x) = (5x + 10)$ CHF, with a fixed cost of 800 CHF.

What will be the cost of producing or rendering 20 units?

R2.11 A certain firm's marginal cost $C'(x)$ and the derivative of the average revenue $\overline{R}'(x)$ are given as follows:

$$C'(x) = (6x + 60) \text{ CHF}$$

$$\overline{R}'(x) = -1 \text{ CHF}$$

If 10 items are produced or rendered, the total costs are 1000 CHF, and the revenue is 1700 CHF.

How many units will result in a maximum profit?
Determine that maximum profit.

R2.12 The supply function for a product or service is

$$p = f_s(x) = (4x + 4) \text{ CHF}$$

and the demand function is

$$p = f_d(x) = (-x^2 + 49) \text{ CHF}$$

Determine the equilibrium point and both the consumer’s and the producer’s surplus there.

R2.13 (see next page)

R2.13 The supply function for a product or a service is

$$p = f_s(x) = \left(ax^2 - \frac{6}{5}x + 2\right) \text{ CHF}$$

and the demand function is

$$p = f_d(x) = (-\,bx^2 + 110) \text{ CHF}$$

with unknown parameters a and b. The equilibrium price is 10 CHF, and the producer's surplus is 73.33 CHF (rounded).

Determine the two unknown parameters a and b.

Hint:
- Use the unrounded value $\left(73 + \frac{1}{3}\right)$ CHF = $\frac{220}{3}$ CHF for the producer's surplus.