Exercises 15   Definite integral
Definite integral, area under a curve, consumer's/producer's surplus

Objectives

- be able to apply the fundamental theorem of calculus.
- be able to determine a definite integral of a constant, basic power, and basic exponential function.
- be able to determine the area between the graph of a basic power function and the abscissa.
- be able to determine a consumer's and a producer's surplus if the demand and supply functions are basic power functions.

Problems

15.1   Calculate the definite integrals below:

a) $\int_3^4 (2x - 5)\,dx$

b) $\int_0^1 (x^3 + 2x)\,dx$

c) $\int_{-5}^{-3} \left(\frac{1}{2}x^2 - 4\right) dx$

d) $\int_2^4 \left(x^3 - \frac{1}{2}x^2 + 3x - 4\right) dx$

e) $\int_{-2}^2 \left(-\frac{1}{8}x^4 + 2x^2\right) dx$

f) $\int_{-1}^1 e^x\,dx$

g) $\int_0^1 e^{2x}\,dx$

h) $\int_{-1}^1 e^{-3x}\,dx$

15.2   Determine the area between the graph of the function f and the x-axis on the interval where the graph of f is above the x-axis, i.e. where $f(x) \geq 0$.

a) $f(x) = -x^2 + 1$

b) $f(x) = x^3 - x^2 - 2x$

Hints:
- First, determine the positions x where the graph of f touches or intersects the x-axis, i.e where $f(x) = 0$
- Then, determine the interval on which the graph of f is above the x-axis, i.e. where $f(x) \geq 0$

15.3   The demand function for a product is $p = f_d(x) = (100 - 4x^2)$ CHF.
If the equilibrium quantity is 4 units, what is the consumer's surplus?

15.4   The demand function for a product is $p = f_d(x) = (34 - x^2)$ CHF.
If the equilibrium price is 9 CHF, what is the consumer's surplus?

15.5   Suppose that the supply function for a good or a service is $p = f_s(x) = (4x^2 + 2x + 2)$ CHF.
If the equilibrium price is 422 CHF, what is the producer's surplus?

15.6   The the supply function f_s and the demand function f_d for a certain product or service are given as follows:

$p = f_s(x) = (x^2 + 4x + 11)$ CHF

$p = f_d(x) = (81 - x^2)$ CHF

Determine …

a)   … the equilibrium point, i.e. the equilibrium quantitiy and the equilibrium price.

b)   … the consumer's surplus at market equilibrium.

c)   … the producer's surplus at market equilibrium.

15.7   (see next page)

15.7   Decide which statements are true or false. Put a mark into the corresponding box.
         In each problem a) to c), exactly one statement is true.

   a)   The definite integral of a function is a ...

          ☐   ... real number.

          ☐   ... function.

          ☐   ... set of functions.

          ☐   ... graph.

   b) $\int_a^b f(x)\,dx$ ...

          ☐   ... = f(b) - f(a)

          ☐   ... = F(a) - F(b) where F is an antiderivative of f.

          ☐   ... is equal to the area between the graph of f and the x-axis on the interval a ≤ x ≤ b if
                f(x) ≥ 0 on the interval a ≤ x ≤ b.

          ☐   ... cannot be calculated unless all antiderivatives of f are known.

   c)   The consumer's surplus is an area between ...

          ☐   ... the graphs of the demand and the supply functions.

          ☐   ... the x axis and the graph of the demand function.

          ☐   ... the graph of the demand function and the horizontal line "price = equilibrium price".

          ☐   ... the horizontal line "price = equilibrium price" and the graph of the supply function.