## Exercises 11   Derivative
Derivative (rate of change), derivative (derived function) of constant/power/exponential functions

Objectives

- be able to estimate a derivative (rate of change) out of the graph of a function.
- be able to state the derivative (rate of change) of a constant and a linear function.
- be able to determine the derivative (derived function) of a constant and a linear function.
- be able to determine the derivative (derived function) of a basic power and a basic exponential function.
- be able to determine a derivative (rate of change) of a basic power and a basic exponential function.

Problems

11.1   The graph of a function f ist given as follows:
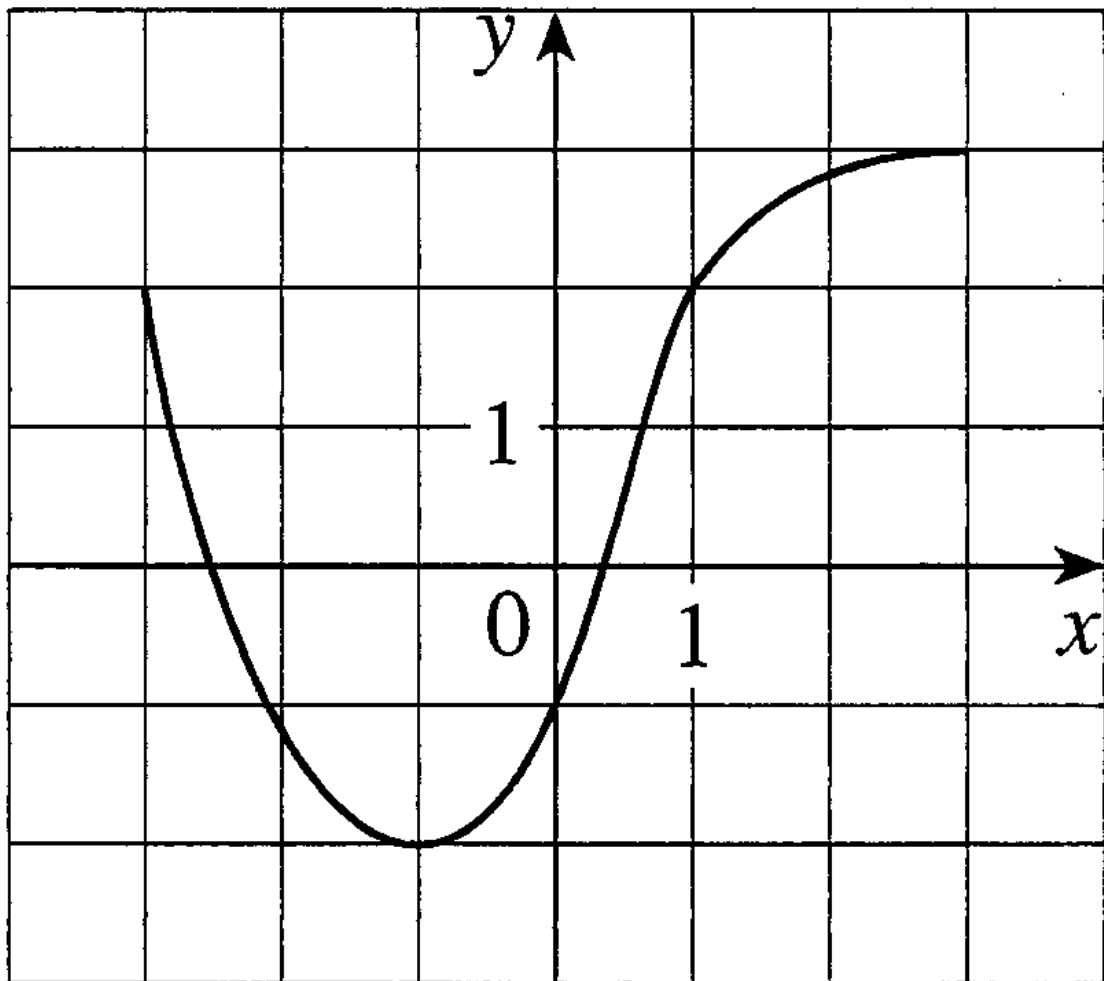
Estimate the derivative (rate of change) $f'(x_0)$ at the given position x_0:

a) $x_0 = -1$ b) $x_0 = 0$

c) $x_0 = 1$ d) $x_0 = -2$

Hints:
- Draw the tangent to the graph of f at the given position x_0.
- Choose any two points on the tangent, and estimate their coordinates.
- Determine the slope of the tangent out of the estimated coordinates of the two points.

11.2   For each of the following functions f: $\mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = ...$

    i)   ... draw the graph of f.

    ii)   ... state the derivative (rate of change) $f'(x_0)$ at the given position x_0.

    a) $f(x) = 3$ $x_0 = 2$

    b) $f(x) = c$ $(c \in \mathbb{R})$ any $x_0 \in \mathbb{R}$

    c) $f(x) = 2x - 3$ $x_0 = 4$

    d) $f(x) = mx + q$ $(m \in \mathbb{R}\setminus\{0\}, q \in \mathbb{R})$ any $x_0 \in \mathbb{R}$

    Hint:
    - If the graph of a function f is a straight line, the derivative (rate of change) $f'(x_0)$ is the slope of that straight
      line, i.e $f'(x_0)$ has the same value at each position x_0, and therefore does not depend on x_0.

11.3   Determine $f'(x)$:

    a) $f(x) = 3$ b) $f(x) = 0$ c) $f(x) = -1$

    d) $f(x) = x^3$ e) $f(x) = x^4$ f) $f(x) = x^5$

    g) $f(x) = x^{17}$ h) $f(x) = x^{200}$ i) $f(x) = x^{100'001}$

    j) $f(x) = x^{-1}$ k) $f(x) = x^{-2}$ l) $f(x) = x^{-17}$

    m) $f(x) = \frac{1}{x}$ n) $f(x) = \frac{1}{x^3}$ o) $f(x) = \frac{1}{x^{99}}$

    p) $f(x) = 3^x$ q) $f(x) = 5^x$ r) $f(x) = \left(\frac{2}{3}\right)^x$

11.4   Determine the derivative (rate of change) $f'(x_0)$ of the function f at the indicated position x_0:

    a) $f(x) = x$

        i) $x_0 = 0$ ii) $x_0 = 1$ iii) $x_0 = -2$

    b) $f(x) = x^5$

        i) $x_0 = 0$ ii) $x_0 = 2$ iii) $x_0 = -\frac{2}{3}$

    c) $f(x) = x^{-4}$

        i) $x_0 = -1$ ii) $x_0 = -\frac{4}{3}$ iii) $x_0 = 0$

    d) $f(x) = \left(\frac{2}{3}\right)^x$

        i) $x_0 = 0$ ii) $x_0 = 1$ iii) $x_0 = -2$

11.5   Decide which statements are true or false. Put a mark into the corresponding box.
    In each problem a) to c), exactly one statement is true.

    a)   The derivative (rate of change) of a function f at the position x_0 is a ...

        ☐   ... real number.
        ☐   ... function.
        ☐   ... tangent.
        ☐   ... graph.

    b)   (see next page)

b) The derivative (derived function) f ' of a function f is a ...

☐ ... real number.

☐ ... function.

☐ ... tangent.

☐ ... graph.

c) $f'(x_0)$ is the slope of the ...

☐ ... secant through the points $(0|0)$ and $(x_0|f(x_0))$.

☐ ... secant through the points $(x_0+\Delta x|f(x_0+\Delta x))$ and $(x_0|f(x_0))$.

☐ ... tangent to the graph of f through $(x_0|f(x_0))$.

☐ ... tangent to the graph of f ' through $(x_0|f(x_0))$.

Answers

11.1 a) $f'(-1) \approx 0$

b) $f'(0) \approx 2$

c) $f'(1) \approx \frac{3}{2}$

d) $f'(-2) \approx -\frac{5}{3}$

11.2 a) i) ...

ii) $f'(2) = 0$

b) i) ...

ii) $f'(x_0) = 0$ at any position x_0

c) i) ...

ii) $f'(4) = 2$

d) i) ...

ii) $f'(x_0) = m$ at any postion x_0

11.3 a) $f'(x) = 0$ b) $f'(x) = 0$ c) $f'(x) = 0$

d) $f'(x) = 3x^2$ e) $f'(x) = 4x^3$ f) $f'(x) = 5x^4$

g) $f'(x) = 17x^{16}$ h) $f'(x) = 200x^{199}$ i) $f'(x) = 100'001x^{100'000}$

j) $f'(x) = -x^{-2}$ k) $f'(x) = -2x^{-3}$ l) $f'(x) = -17x^{-18}$

m) $f'(x) = -\frac{1}{x^2}$ n) $f'(x) = -\frac{3}{x^4}$ o) $f'(x) = -\frac{99}{x^{100}}$

p) $f'(x) = 3^x \ln(3)$ q) $f'(x) = 5^x \ln(5)$ r) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$

11.4 a) $f'(x) = 1$

i) $f'(0) = 1$ ii) $f'(1) = 1$ iii) $f'(-2) = 1$

b) $f'(x) = 5x^4$

i) $f'(0) = 0$ ii) $f'(2) = 80$ iii) $f'\left(-\frac{2}{3}\right) = \frac{80}{81}$

c) $f'(x) = -\frac{4}{x^5}$

i) $f'(-1) = 4$ ii) $f'\left(-\frac{4}{3}\right) = \frac{243}{256}$ iii) $f'(0)$ is not defined
(division by zero)

d) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$

i) $f'(0) = \ln\left(\frac{2}{3}\right)$ ii) $f'(1) = \frac{2}{3}\ln\left(\frac{2}{3}\right)$ iii) $f'(-2) = \frac{9}{4}\ln\left(\frac{2}{3}\right)$

11.5 a) 1st statement

b) 2nd statement

c) 3rd statement