

## Exercises 4                      Linear function and equations Linear equations

### Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to perform a case differentiation.
- be able to treat applied tasks in economics by means of linear equations.

### Problems

4.1 Determine the solution sets of the following equations:

- a)  $19x - 32 + 17x = 18x - 30 + 16x - 4$
- b)  $25x - 16 - 9x = 20 + 24x - 10 - 10x$
- c)  $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$
- d)  $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

4.2 Determine the solution sets of the following equations:

- a)  $22(x - 11) - 5(x - 40) = 110 - (x + 53)$
- b)  $184 - 6(x - 24) = 214 - 3(2x - 38)$
- c)  $(x - 5)(x - 2) = (x - 4)(x - 3)$
- d)  $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

4.3 Determine the solution sets of the following equations:

- a)  $\frac{x+3}{5} = \frac{2x-8}{3}$
- b)  $\frac{x+3}{4} + \frac{1-3x}{7} = 0$
- c)  $\frac{2}{x-1} = \frac{1}{x-2}$
- d)  $\frac{x}{x-1} = \frac{x-1}{x-2}$

4.4 The equations below are equations in the variable  $x$ . Furthermore, the equations contain parameters  $a$  and  $b$ . Therefore, the solution sets of the equations depend on the values of those parameters.

Solve the equations for  $x$ , and determine the solution sets.

Take into account that the parameters  $a$  and  $b$  can be any real numbers.

a)  $x(a - 3) = a$

Hints:

- You may want to divide both sides of the equation by  $a - 3$ . However, this is not allowed if  $a - 3 = 0$ , i.e. if  $a = 3$ , as dividing by 0 is not defined.
- When solving the equation, consider the two cases  $a \neq 3$  and  $a = 3$ .

b)  $(x + 1)(b - 2) = 2bx$

c)  $(a - b)x = a$

4.5 The graph of a linear function  $f$  with slope  $a$  contains the point  $P$ . Find the equation of the linear function.

- a)  $a = -5$                        $P(5|-3)$
- b)  $a = 2$                           $P(3|0)$
- c)  $a = 0$                           $P(2|3)$

4.6 Alps Bikes uses the formula  $B(t) = (-400t + 5000)$  CHF to find the book value  $B(t)$  of a mountain bike after the time  $t$  after its purchase ( $t =$  number of years after the purchase).

- a) What do the numbers  $-400$  and  $5000$  signify?
- b) How long will it take the mountain bike to depreciate completely?

4.7 Two items A and B depreciate linearly:

- Item A                      original value = 200 CHF  
                                  depreciation = 16 CHF/year
- Item B                      original value = 240 CHF  
                                  depreciation = 32 CHF/year

- a) How long will it take the two items to depreciate completely?
- b) After how much time will the two items have the same value?

Hint:

- Think of the temporal development of the values as linear functions.

4.8 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.

- a) The solution set of a linear equation ...
  - ... always contains at least one element.
  - ... never contains two elements.
  - ... only contains elements if the linear equation corresponds to a constant function.
  - ... cannot be the empty set.
- b) If a linear equation has exactly one solution ...
  - ... the graph of the corresponding linear function intersects the x-axis.
  - ... the equation does not contain any parameters.
  - ... the solution must be an integer.
  - ... no other linear equation can have the same solution.
- c) If a linear equation has the solution  $x = 2$ , it can be concluded that ...
  - ...  $x = 3$  is not a solution.
  - ... the graph of the corresponding linear function intersects the x-axis at  $x = 2$ .
  - ...  $P(2|0)$  is a point of the graph of the corresponding linear function.
  - ...  $P(0|2)$  is a point of the graph of the corresponding linear function.

**Answers**

- 4.1 a)  $S = \{-1\}$       b)  $S = \{13\}$   
 c)  $S = \mathbb{R}$       d)  $S = \{ \}$

- 4.2 a)  $S = \left\{ \frac{11}{2} \right\}$       b)  $S = \mathbb{R}$   
 c)  $S = \{ \}$       d)  $S = \{0\}$

- 4.3 a)  $S = \{7\}$       b)  $S = \{5\}$   
 c)  $S = \{3\}$       d)  $S = \{ \}$

- 4.4 a) if  $a = 3$ : no solution  $\Rightarrow S = \{ \}$   
 if  $a \neq 3$ :  $x = \frac{a}{a-3}$   $\Rightarrow S = \left\{ \frac{a}{a-3} \right\}$   
 b) if  $b = -2$ : no solution  $\Rightarrow S = \{ \}$   
 if  $b \neq -2$ :  $x = \frac{b-2}{b+2}$   $\Rightarrow S = \left\{ \frac{b-2}{b+2} \right\}$   
 c) if  $a = b = 0$ :  $x \in \mathbb{R}$   $\Rightarrow S = \mathbb{R}$   
 if  $a = b \neq 0$ : no solution  $\Rightarrow S = \{ \}$   
 if  $a \neq b$ :  $x = \frac{a}{a-b}$   $\Rightarrow S = \left\{ \frac{a}{a-b} \right\}$

- 4.5 a)  $y = f(x) = -5x + 22$

Hints:

- The equation of a linear function is  $y = f(x) = ax + b$
- $P(5|-3)$  is a point of the graph of the linear function. Therefore, the coordinates of P must fulfil the equation of the linear function, i.e.  $-3 = f(5) = a \cdot 5 + b$

- b)  $y = f(x) = 2x - 6$

- c)  $y = f(x) = 3$

- 4.6 a) The number - 400 indicates that the value of the mountain bike decreases by 400 CHF per year. The number 5000 indicates that the original value of the mountain bike was 5000 CHF.

- b) 12.5 years

Hint:

- Complete depreciation at time t means  $B(t) = 0$ .

- 4.7 a) item A: 12.5 years  
 item B: 7.5 years

- b) 2.5 years

- 4.8 a) 2<sup>nd</sup> statement

- b) 1<sup>st</sup> statement

- c) 3<sup>rd</sup> statement