

## Exercises 1                      Number sets N, Z, Q, R, set operations

### Objectives

- know the definition and elements of natural numbers, integers, rational numbers, and real numbers.
- know and understand what a set, an element of a set, an empty set, a subset, an intersection, a union, and a set difference are.
- be able to perform basic set operations.

### Problems

1.1     Decide whether each statement is true or false:

- |                                    |  |   |
|------------------------------------|--|---|
| a) $4 \in \mathbb{N}$              | b) $-\frac{14}{7} \in \mathbb{Z}$      | c) $\sqrt{2} \in \mathbb{Q}$                      |
| d) $\sqrt{9} \in \mathbb{N}$       | e) $\sqrt{9} \in \mathbb{Q}$           | f) $\sqrt{9} \in \mathbb{R}$                      |
| g) $1.67854 \in \mathbb{Q}$        | h) $1.678\overline{54} \in \mathbb{Q}$ | i) $\mathbb{N} \subset \mathbb{Z}$                |
| j) $\mathbb{Z} \subset \mathbb{Q}$ | k) $\mathbb{Q} \subset \mathbb{R}$     | l) $\mathbb{R} \setminus \mathbb{Z} = \mathbb{N}$ |

1.2     Determine the following sets:

- |  |  |  |
|--|--|--|
| a) $\mathbb{Z} \setminus \mathbb{N}$                   | b) $\mathbb{Z} \cup \mathbb{N}$                        | c) $\mathbb{Z} \cap \mathbb{N}$                        |
| d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$ | e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$ | f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N}$ |

1.3     Look at the sets A, B, and C:

A = Set of all cities of the world  
B = Set of all European cities  
C = Set of all coastal cities of the world

Find at least five elements of the following sets:

- |                    |                             |
|--------------------|-----------------------------|
| a) $B \cap C$      | b) $B \setminus C$          |
| c) $C \setminus B$ | d) $A \setminus (B \cup C)$ |

1.4     Decide which statements are true or false. Put a mark into the corresponding box.  
In each problem a) to c), exactly one statement is true.

- a)       $\mathbb{N} \cup \mathbb{Z} = \mathbb{Q}$   
  $\mathbb{Q} \setminus \mathbb{Z} = \mathbb{N}$   
  $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$   
  $\mathbb{Z} \setminus \mathbb{N} = \{-1, -2, -3, \dots\}$

b)     A = Set of all cities of the world  
       B = Set of all European cities

- $A \cap B = A$   
  $A \cup B = B$   
  $B \in A$   
  $B \subset A$

c)     (see next page)

c) Assume that  $x$  is a rational number. Therefore, it can be concluded that  $x$  is ...

- ... a real number.
- ... an integer.
- ... a fraction where both numerator and denominator are natural numbers.
- ... a natural number.