Exercises 1 Number sets
ℕ, ℤ, ℚ, ℝ, set operations

Objectives

- know the definition and elements of natural numbers, integers, rational numbers, and real numbers.
- know and understand what a set, an element of a set, an empty set, a subset, an intersection, a union, and a set difference are.
- be able to perform basic set operations.

Problems

1.1   Decide whether each statement is true or false:

a) $4 \in \mathbb{N}$ b) $-\frac{14}{7} \in \mathbb{Z}$ c) $\sqrt{2} \in \mathbb{Q}$

d) $\sqrt{9} \in \mathbb{N}$ e) $\sqrt{9} \in \mathbb{Q}$ f) $\sqrt{9} \in \mathbb{R}$

g) $1.67854 \in \mathbb{Q}$ h) $1.67\overline{854} \in \mathbb{Q}$ i) $\mathbb{N} \subset \mathbb{Z}$

j) $\mathbb{Z} \subset \mathbb{Q}$ k) $\mathbb{Q} \subset \mathbb{R}$ l) $\mathbb{R} \setminus \mathbb{Z} = \mathbb{N}$

1.2   Determine the following sets:

a) $\mathbb{Z} \setminus \mathbb{N}$ b) $\mathbb{Z} \cup \mathbb{N}$ c) $\mathbb{Z} \cap \mathbb{N}$

d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$ e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$ f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N}$

1.3   Look at the sets A, B, and C:

A = Set of all cities of the world
B = Set of all European cities
C = Set of all coastal cities of the world

Find at least five elements of the following sets:

a) B ∩ C   b) B \ C

c) C \ B   d) A \ (B ∪ C)

1.4   Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) ☐ $\mathbb{N} \cup \mathbb{Z} = \mathbb{Q}$
   ☐ $\mathbb{Q} \setminus \mathbb{Z} = \mathbb{N}$
   ☐ $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$
   ☐ $\mathbb{Z} \setminus \mathbb{N} = \{-1, -2, -3, ...\}$

b) A = Set of all cities of the world
   B = Set of all European cities

   ☐ A ∩ B = A
   ☐ A ∪ B = B
   ☐ B ∈ A
   ☐ B ⊂ A

c) (see next page)

c) Assume that x is a rational number. Therefore, it can be concluded that x is ...

☐   ... a real number.

☐   ... an integer.

☐   ... a fraction where both numerator and denominator are natural numbers.

☐   ... a natural number.

Answers

1.1   a)   true   b)   true   c)   false

         d)   true   e)   true   f)   true

         g)   true   h)   true   i)   true

         j)   true   k)   true   l)   false

1.2   a) $\mathbb{Z} \setminus \mathbb{N} = \{0, -1, -2, -3, ...\}$

         b) $\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$

         c) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$

         d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q}) = \{\}$

         e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$

         f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N} = \{\}$

1.3   a)   B ∩ C = {Lisbon, Copenhagen, Barcelona, Naples, Stockholm, ...}

         b)   B \ C = {London, Paris, Madrid, Berlin, Rome, ...}

         c)   C \ B = {Tokyo, San Francisco, Sydney, Rio de Janeiro, Cape Town, ...}

         d)   A \ (B ∪ C) = {Chicago, Mexico City, Nairobi, Beijing, Bogotá, ...}

1.4   a)   3rd statement

         b)   4th statement

         c)   1st statement