## Function

## Definition and examples

Def.: A function f is a rule that assigns to each element x in a set D exactly one element y in a set B .


The function f maps the set D onto the set B .

$$
\begin{aligned}
\mathrm{f}: & \mathrm{D} \rightarrow \mathrm{~B} \\
\mathrm{x} & \rightarrow \mathrm{y}=\mathrm{f}(\mathrm{x}) \quad \text { ("f of } \mathrm{x} ")
\end{aligned}
$$

The set D is the domain, the set B is the codomain, and the set E is the range of the function f .
The element y is the image of the element x .
or (if $D$ and $B$ are number sets): $y$ is the value of $f$ at $x$.


Ex.: 1. $\quad \mathrm{D}=$ set of all Swiss holiday resorts
$B=$ set of all human beings
f: $\quad \mathrm{D} \rightarrow \mathrm{B}$
$r \rightarrow d=f(r)=$ director of holiday resort $r$ in 2000

2. $\mathrm{D}=$ set of all countries of the world

B = set of all cities of the world

$$
\begin{array}{ll}
\text { f: } \quad & \mathrm{D} \rightarrow \mathrm{~B} \\
\mathrm{a} \rightarrow \mathrm{~b}=\mathrm{f}(\mathrm{a})=\text { capital of country a }
\end{array}
$$

3. Cable car company
$\mathrm{D}=\mathbb{N} \quad$ (= set of natural numbers)
$B=\mathbb{R} \quad$ (= set of real numbers)
f: $\quad \mathrm{D} \rightarrow \mathrm{B}$
$n \rightarrow r=f(n)=$ revenue (e.g. in Euros) when $n$ tickets are sold
4. $\mathrm{D}=\mathbb{N}$
$B=\mathbb{Z}$
f: $\quad \mathbb{N} \rightarrow \mathbb{Z}$
$\mathrm{n} \rightarrow \mathrm{y}=\mathrm{f}(\mathrm{n})=\mathrm{n}-4$

5. $\mathrm{D}=\mathrm{B}=\mathbb{R}$
$\mathrm{p}: \quad \mathbb{R} \rightarrow \mathbb{R}$

$$
x \rightarrow y=p(x)=\frac{x^{3}-3}{2 x^{2}+1}
$$

## Representation of a function

## Arrow diagram



Table of values

| x | y |
| :---: | :---: |
| 1 | 1 |
| 3 | 9 |
| 5 | 25 |
| -5 | 25 |
| 0.4 | 0.16 |
| $\ldots$ | $\ldots$ |

Equation

$$
\text { f: } \begin{aligned}
& \mathbb{R} \rightarrow \mathbb{R} \\
& x \rightarrow y=f(x)=x^{2}
\end{aligned}
$$

## Graph



