

# Formulary

## 1. Algebra

### Binomial formulae

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

### Powers and roots

$$a^n \cdot a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{n \cdot m} = a^{m \cdot n} = (a^m)^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = \sqrt[n]{a^m}$$

$$a^n \cdot b^n = (a \cdot b)^n$$

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

## 2. Logarithms

$$a^x = y \Leftrightarrow x = \log_a(y)$$

$$\log_a(u \cdot v) = \log_a(u) + \log_a(v)$$

$$\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$$

$$\log_a(u^v) = v \cdot \log_a(u)$$

## 3. Functions and equations

### Linear function

$$y = f(x) = ax + b$$

$$\text{slope } a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

### Quadratic function (graph = parabola)

$$y = f(x) = ax^2 + bx + c$$

general form

$$y = f(x) = a(x - u)^2 + v$$

vertex form, vertex  $V(u|v)$

### Quadratic equation

$$ax^2 + bx + c = 0$$

general form

$$\Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Exponential function

$$y = f(x) = c \cdot a^x$$

$a$  = growth factor

## 4. Financial mathematics

$r$	interest rate per compounding period
$n$	number of compounding periods
$q$	interest/growth factor, $q = 1 + r$
$C_0$	initial capital
$C_n$	capital after $n$ compounding periods
Ordinary annuity	payments are made at the <b>end</b> of the compounding period
Annuity due	payments are made at the <b>beginning</b> of the compounding period
$p$	payment before/after each compounding period (all payments are equal)
$A_n$	value of an annuity after $n$ compounding periods (starting with $C_0 = 0$ )
$A_0$	initial value of an annuity (ending with $C_n = 0$ )

### Interest

#### Simple interest

$$C_n = C_0 (1 + nr) \quad \Rightarrow \quad C_0 = \frac{C_n}{1 + nr} \quad r = \frac{\frac{C_n}{C_0} - 1}{n} \quad n = \frac{\frac{C_n}{C_0} - 1}{r}$$

#### Compound interest

$$C_n = C_0 q^n \quad \Rightarrow \quad C_0 = \frac{C_n}{q^n} \quad q = \sqrt[n]{\frac{C_n}{C_0}} \quad n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{\log_a(q)}$$

$$C_n = C_0 (1 + r)^n \quad \Rightarrow \quad C_0 = \frac{C_n}{(1 + r)^n} \quad r = \sqrt[n]{\frac{C_n}{C_0}} - 1 \quad n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{\log_a(1 + r)}$$

### Annuity

#### Ordinary annuity

$$A_n = p \frac{q^n - 1}{q - 1} \quad \Rightarrow \quad p = \frac{A_n(q - 1)}{q^n - 1} \quad n = \frac{\log_a\left(\frac{A_n(q - 1)}{p} + 1\right)}{\log_a(q)}$$

$$A_0 = p \frac{q^n - 1}{q^n(q - 1)} \quad \Rightarrow \quad p = \frac{A_0 q^n (q - 1)}{q^n - 1} \quad n = \frac{\log_a\left(\frac{p}{p - A_0(q - 1)}\right)}{\log_a(q)}$$

#### Annuity due

$$A_n = pq \frac{q^n - 1}{q - 1} \quad \Rightarrow \quad p = \frac{A_n(q - 1)}{q(q^n - 1)} \quad n = \frac{\log_a\left(\frac{A_n(q - 1)}{pq} + 1\right)}{\log_a(q)}$$

$$A_0 = p \frac{q^n - 1}{q^{n-1}(q - 1)} \quad \Rightarrow \quad p = \frac{A_0 q^{n-1} (q - 1)}{q^n - 1} \quad n = \frac{\log_a\left(\frac{pq}{pq - A_0(q - 1)}\right)}{\log_a(q)}$$

## 5. Differential calculus

Derivative	$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$	
Constant function	$f(x) = c \quad (c \in \mathbb{R})$	$\Rightarrow f'(x) = 0$
Power function	$f(x) = x^n \quad (n \in \mathbb{R})$	$\Rightarrow f'(x) = n \cdot x^{n-1}$
Exponential function	$f(x) = a^x \quad (a \in \mathbb{R}^+ \setminus \{1\})$	$\Rightarrow f'(x) = a^x \cdot \ln(a)$
	$f(x) = e^x$	$\Rightarrow f'(x) = e^x$
	$f(x) = e^{f_1(x)}$	$\Rightarrow f'(x) = f_1'(x) \cdot e^{f_1(x)}$
Coefficient rule	$f(x) = c \cdot f_1(x) \quad (c \in \mathbb{R})$	$\Rightarrow f'(x) = c \cdot f_1'(x)$
Sum rule	$f(x) = f_1(x) \pm f_2(x)$	$\Rightarrow f'(x) = f_1'(x) \pm f_2'(x)$
Product rule	$f(x) = f_1(x) \cdot f_2(x)$	$\Rightarrow f'(x) = f_1'(x) \cdot f_2(x) + f_1(x) \cdot f_2'(x)$

## 6. Integral calculus

Constant function	$\int c \, dx = cx + C$	$(c \in \mathbb{R}, C \in \mathbb{R})$
Power function	$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$	$(n \in \mathbb{R} \setminus \{-1\}, C \in \mathbb{R})$
	$\int x^{-1} \, dx = \ln( x ) + C$	$(C \in \mathbb{R})$
Exponential function	$\int e^x \, dx = e^x + C$	$(C \in \mathbb{R})$
	$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C$	$(k \in \mathbb{R} \setminus \{0\}, C \in \mathbb{R})$
Coefficient rule	$\int c \cdot f(x) \, dx = c \cdot \int f(x) \, dx$	$(c \in \mathbb{R})$
Sum rule	$\int (f_1(x) \pm f_2(x)) \, dx = \int f_1(x) \, dx \pm \int f_2(x) \, dx$	
Definite integral	$\int_a^b f(x) \, dx = [F(x)]_a^b = F(b) - F(a)$	