

## Exercises 9                      Exponential function and equations Exponential equations, logarithm, compound interest

### Objectives

- be able to determine simple logarithms without a calculator.
- be able to solve simple exponential equations without a calculator.
- be able to calculate a common and a natural logarithm with a calculator.
- be able to apply one of the logarithmic properties in order to solve simple exponential equations.
- be able to treat specific compound interest tasks by means of logarithms.

### Problems

#### *Exponential equations, logarithm*

9.1 Solve the exponential equations below **without** a calculator, i.e. find the solutions by guessing.

- |  |                       |                |
|--|-----------------------|----------------|
| a) $2^x = 16$                                  | b) $4^x = 64$         | c) $5^x = 1$   |
| d) $\left(\frac{3}{2}\right)^x = \frac{27}{8}$ | e) $10^x = 1'000'000$ | f) $10^x = 10$ |

9.2 Determine the following logarithms **without** a calculator.

- |                      |                   |                  |
|----------------------|-------------------|------------------|
| a) $\log_3(27)$      | b) $\log_4(16)$   | c) $\log_2(128)$ |
| d) $\log_{10}(1000)$ | e) $\log_{10}(1)$ |                  |

9.3 Determine the logarithms below **with** your calculator.

- |               |                   |                   |
|---------------|-------------------|-------------------|
| a) $\lg(1.1)$ | b) $\ln(1.1)$     | c) $\lg(9)$       |
| d) $\ln(9)$   | e) $\lg(2345.67)$ | f) $\ln(2345.67)$ |

9.4 Solve the following exponential equations.

- |                |                    |                       |
|----------------|--------------------|-----------------------|
| a) $10^x = 21$ | b) $10^x = 256.78$ | c) $10^x = 1'234'567$ |
|----------------|--------------------|-----------------------|

9.5 Solve the exponential equations below.

- |                    |                       |                  |
|--------------------|-----------------------|------------------|
| a) $3^x = 99$      | b) $1.01^x = 1.5$     | c) $3^{x+4} = 5$ |
| d) $5^{2x-1} = 12$ | e) $1 - e^{5x} = 0.3$ |                  |

#### *Compound interest*

9.6 An initial capital  $C_0$  is invested at an annual interest rate  $r$ , compounded annually. After  $n$  years the capital amounts to  $C_n$ . Determine  $n$ .

- |                        |              |                               |
|------------------------|--------------|-------------------------------|
| a) $C_0 = 1000$ CHF    | $r = 1.00\%$ | $C_n = 1220$ CHF (rounded)    |
| b) $C_0 = 100'000$ CHF | $r = 2.25\%$ | $C_n = 243'519$ CHF (rounded) |

9.7 How long would 10'000 CHF have to be invested at an annual interest rate of 2.5%, compounded annually, to amount to 12'000 CHF?

- 9.8 How long would any initial capital have to be invested at an annual interest rate of 1.25%, compounded annually, to double its value?
- 9.9 An initial capital of 10'000.00 CHF is invested at an unknown annual interest rate, compounded annually. After 10 years the capital amounts to 11'894.40 CHF. After how many years (from the beginning of the investment) will the capital be worth 15'000.00 CHF?
- 9.10 How long (in years) would 1000 CHF have to be invested at a nominal annual interest rate of 2.5%, compounded daily, to earn 250 CHF total interest?
- 9.11 A couple needs 150'000 CHF as a down payment for a home. If they invest the 100'000 CHF they have at a nominal annual interest rate of 8%, compounded quarterly, how long will it take for the money to grow into 150'000 CHF?
- 9.12 The sales decay for a product is given by
- $$S = 50'000 e^{-0.8x} \text{ CHF}$$
- where S is the monthly sales and x is the number of months that have passed since the end of a promotional campaign.
- a) What will be the sales 4 months after the end of the campaign?
- b) After how many months after the end of the campaign will sales have dropped below 1000 CHF, if no new campaign is initiated?
- 9.13 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.
- a)  $\log_a(x)$  is the answer to the question ...
- ... “a to what power is x?”
- ... “x to what power is a?”
- ... “10 to what power is x?”
- ... “10 to what power is a?”
- b) If  $p = 2^q$  then ...
- ...  $q = \left(\frac{1}{2}\right)^p$
- ...  $q = \frac{p}{2}$
- ...  $q = \log_2(p)$
- ...  $q = \ln(2)$
- c)  $\ln(e) = \dots$
- ...  $\log_e(1)$
- ...  $\log_{10}(e)$
- ... 0
- ... 1