

Exercises 7 Quadratic function and equations Quadratic function/equations, supply, demand, market equilibrium

Objectives

- know and understand the relation between a quadratic function and a quadratic equation.
- be able to solve a quadratic equation with the method of completing the square.
- be able to solve a quadratic equation by applying the quadratic formula.
- be able to solve special quadratic equations without applying the quadratic formula.
- be able to solve a quadratic equation containing a parameter.
- be able to determine the vertex form of the equation of a quadratic function out of the coordinates of the vertex and the coordinates of another point of the corresponding parabola.
- be able to determine the general form of the equation of a quadratic function out of the coordinates of three points of the corresponding parabola.
- be able to treat applied tasks in economics by means of quadratic equations or systems of quadratic equations.

Problems

7.1 Each quadratic equation can be converted into the following general form:

$$ax^2 + bx + c = 0 \quad (a \in \mathbb{R} \setminus \{0\}, b \in \mathbb{R}, c \in \mathbb{R}) \quad (*)$$

Determine the number of solutions that a quadratic equation can have, i.e. try to find out the different possible cases of the number of solutions.

Hints:

- Remember our discussion about the possible number of solutions of a linear equation.
- Compare the left hand side of the quadratic equation (*) with the general form of the equation of a quadratic function.
- Think of the graph of a quadratic function.

7.2 Solve the quadratic equations below using ...

- i) ... the method of completing the square.
- ii) ... the quadratic formula.

State the solution set for each equation.

- a) $x^2 + 10x + 24 = 0$
- b) $2x^2 - 7x + 3 = 0$
- c) $x^2 + 2x + 8 = 0$
- d) $x^2 - 14x + 49 = 0$

7.3 Solve the quadratic equations below using the quadratic formula. State the solution set for each equation.

- a) $x^2 + 22x + 121 = 0$
- b) $5x^2 + 8x - 4 = 0$
- c) $5x^2 - 8x + 4 = 0$
- d) $24x^2 - 65x + 44 = 0$
- e) $\frac{1}{6}x^2 - \frac{5}{4}x + \frac{3}{2} = 0$
- f) $-9x^2 - 54x - 63 = 0$

7.4 Solve the equations below. State the solution set for each equation.

- a) $9(x - 10) - x(x - 15) = x$
- b) $3(x^2 + 2) - x(x + 9) = 11$
- c) $y^3 + 19 = (y + 4)^3$
- d) $\frac{9x - 8}{4x + 7} = \frac{3x}{2x + 5}$
- e) $\frac{x^2}{x - 6} - \frac{6x}{6 - x} = 1$
- f) $\frac{8}{x^2 - 4} + \frac{2}{2 - x} = 3x - 1$

7.5 Solve the quadratic equations below without using the quadratic formula. State the solution set for each equation.

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|----|----------------------|----|-----------------------|
| a) | $(x + 2)(x + 5) = 0$ | b) | $(x - 8)(5x - 9) = 0$ |
| c) | $x^2 - 3x = 0$ | d) | $x^2 + 7x = 0$ |
| e) | $4x^2 - 9 = 0$ | f) | $100x^2 - 1 = 0$ |
| g) | $3x^2 = 27$ | h) | $x^2 = x$ |

7.6 Solve the equations below. State the solution set for each equation.

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|----|--|----|---------------------------|
| a) | $(7 + x)(7 - x) = (3x + 2)^2 - (2x + 3)^2$ | b) | $(x - 3)(2x - 7) = 1$ |
| c) | $\frac{x-4}{x-5} = \frac{30-x^2}{x^2-5x}$ | d) | $\frac{x^2-x-2}{2-x} = 1$ |
| e) | $\frac{x^2-4}{x^2-4} = 0$ | f) | $\frac{x^2-4}{x^2-4} = 1$ |

7.7 The quadratic equations below contain a parameter p . Therefore, the solution set of the equations will depend on the value of this parameter.

Solve the equations for x .

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|----|-------------------|----|---------------------|
| a) | $x^2 + x + p = 0$ | b) | $3x^2 + px - p = 0$ |
|----|-------------------|----|---------------------|

7.8 A parabola has the vertex V and contains the point P .

Determine the equation of the corresponding quadratic function both in the vertex and in the general form.

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|----|-----------|-----------|
| a) | $V(2 4)$ | $P(-1 7)$ |
| b) | $V(1 -8)$ | $P(2 -7)$ |

7.9 A parabola contains the three points P , Q , and R .

Determine the equation of the corresponding quadratic function in the general form.

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|----|-----------|----------|------------|
| a) | $P(-4 8)$ | $Q(0 0)$ | $R(10 15)$ |
| b) | $P(1 -1)$ | $Q(2 4)$ | $R(4 8)$ |

7.10 Find the equilibrium quantity and equilibrium price of a service for the given supply and demand functions f_s and f_d :

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|----|--------|---|
| a) | supply | $p = f_s(q) = \left(\frac{1}{4}q^2 + 10\right) \text{ CHF}$ |
| | demand | $p = f_d(q) = (86 - 6q - 3q^2) \text{ CHF}$ |
| b) | supply | $p = f_s(q) = (q^2 + 8q + 16) \text{ CHF}$ |
| | demand | $p = f_d(q) = (-3q^2 + 6q + 436) \text{ CHF}$ |

7.11 The total costs $C(x)$ for producing x items and the revenues $R(x)$ for selling x items are given by

$$C(x) = (2000 + 40x + x^2) \text{ CHF}$$

$$R(x) = 130x \text{ CHF}$$

Find the break-even values of x .

7.12 The total costs $C(x)$ for producing x items and the revenues $R(x)$ for selling x items are given by

$$C(x) = (x^2 + 100x + 80) \text{ CHF}$$

$$R(x) = (160x - 2x^2) \text{ CHF}$$

How many items are to be produced and sold in order to achieve a profit of 200 CHF?

7.13 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) A quadratic equation ...

- ... has no solution whenever the vertex of the graph of the corresponding quadratic function is below the x-axis.
- ... always has one or two solutions.
- ... has exactly one solution if the vertex of the graph of the corresponding quadratic function is on the x-axis.
- ... can have infinitely many solutions.

b) The graph of a quadratic function ...

- ... is uniquely defined whenever the vertex and one further point of the graph are known.
- ... is a straight line if the corresponding quadratic equation has exactly one solution.
- ... is a quadratic equation.
- ... can be determined by solving a quadratic equation.

c) If the total cost function is quadratic and the total revenue function is linear ...

- ... there is always exactly one break-even point.
- ... a break-even point corresponds to a solution of a quadratic equation.
- ... no profit can be realised whenever the linear function has a positive slope.
- ... the vertex of the graph of the cost function cannot be below the x-axis.