

Exercises 4 **Linear function and equations** **Linear equations**

Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to perform a case differentiation.
- be able to treat applied tasks in economics by means of linear equations.

Problems

4.1 Determine the solution sets of the following equations:

- a) $19x - 32 + 17x = 18x - 30 + 16x - 4$
- b) $25x - 16 - 9x = 20 + 24x - 10 - 10x$
- c) $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$
- d) $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

4.2 Determine the solution sets of the following equations:

- a) $22(x - 11) - 5(x - 40) = 110 - (x + 53)$
- b) $184 - 6(x - 24) = 214 - 3(2x - 38)$
- c) $(x - 5)(x - 2) = (x - 4)(x - 3)$
- d) $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

4.3 Determine the solution sets of the following equations:

- a) $\frac{x+3}{5} = \frac{2x-8}{3}$
- b) $\frac{x+3}{4} + \frac{1-3x}{7} = 0$
- c) $\frac{2}{x-1} = \frac{1}{x-2}$
- d) $\frac{x}{x-1} = \frac{x-1}{x-2}$

4.4 The equations below are equations in the variable x . Furthermore, the equations contain parameters a and b . Therefore, the solution sets of the equations depend on the values of those parameters.

Solve the equations for x , and determine the solution sets.

Take into account that the parameters a and b can be any real numbers.

a) $x(a - 3) = a$

Hints:

- You may want to divide both sides of the equation by $a - 3$. However, this is not allowed if $a - 3 = 0$, i.e. if $a = 3$, as dividing by 0 is not defined.
- When solving the equation, consider the two cases $a \neq 3$ and $a = 3$.

b) $(x + 1)(b - 2) = 2bx$

c) $(a - b)x = a$

4.5 The graph of a linear function f with slope a contains the point P . Find the equation of the linear function.

- a) $a = -5$ $P(5|-3)$
- b) $a = 2$ $P(3|0)$
- c) $a = 0$ $P(2|3)$

4.6 Alps Bikes uses the formula $B(t) = (-400t + 5000)$ CHF to find the book value $B(t)$ of a mountain bike after the time t after its purchase (t = number of years after the purchase).

- a) What do the numbers -400 and 5000 signify?
- b) How long will it take the mountain bike to depreciate completely?

4.7 Two items A and B depreciate linearly:

- Item A original value = 200 CHF
 depreciation = 16 CHF/year
- Item B original value = 240 CHF
 depreciation = 32 CHF/year

- a) How long will it take the two items to depreciate completely?
- b) After how much time will the two items have the same value?

Hint:

- Think of the temporal development of the values as linear functions.

4.8 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.

- a) The solution set of a linear equation ...
 - ... always contains at least one element.
 - ... never contains two elements.
 - ... only contains elements if the linear equation corresponds to a constant function.
 - ... cannot be the empty set.
- b) If a linear equation has exactly one solution ...
 - ... the graph of the corresponding linear function intersects the x-axis.
 - ... the equation does not contain any parameters.
 - ... the solution must be an integer.
 - ... no other linear equation can have the same solution.
- c) If a linear equation has the solution $x = 2$, it can be concluded that ...
 - ... $x = 3$ is not a solution.
 - ... the graph of the corresponding linear function intersects the x-axis at $x = 2$.
 - ... $P(2|0)$ is a point of the graph of the corresponding linear function.
 - ... $P(0|2)$ is a point of the graph of the corresponding linear function.

Answers

- 4.1 a) $S = \{-1\}$ b) $S = \{13\}$
 c) $S = \mathbb{R}$ d) $S = \{\}$

- 4.2 a) $S = \left\{\frac{11}{2}\right\}$ b) $S = \mathbb{R}$
 c) $S = \{\}$ d) $S = \{0\}$

- 4.3 a) $S = \{7\}$ b) $S = \{5\}$
 c) $S = \{3\}$ d) $S = \{\}$

- 4.4 a) if $a = 3$: no solution $\Rightarrow S = \{\}$
 if $a \neq 3$: $x = \frac{a}{a-3}$ $\Rightarrow S = \left\{\frac{a}{a-3}\right\}$
 b) if $b = -2$: no solution $\Rightarrow S = \{\}$
 if $b \neq -2$: $x = \frac{b-2}{b+2}$ $\Rightarrow S = \left\{\frac{b-2}{b+2}\right\}$
 c) if $a = b = 0$: $x \in \mathbb{R}$ $\Rightarrow S = \mathbb{R}$
 if $a = b \neq 0$: no solution $\Rightarrow S = \{\}$
 if $a \neq b$: $x = \frac{a}{a-b}$ $\Rightarrow S = \left\{\frac{a}{a-b}\right\}$

- 4.5 a) $y = f(x) = -5x + 22$

Hints:

- The equation of a linear function is $y = f(x) = ax + b$
- $P(5|-3)$ is a point of the graph of the linear function. Therefore, the coordinates of P must fulfil the equation of the linear function, i.e. $-3 = f(5) = a \cdot 5 + b$

- b) $y = f(x) = 2x - 6$

- c) $y = f(x) = 3$

- 4.6 a) The number - 400 indicates that the value of the mountain bike decreases by 400 CHF per year. The number 5000 indicates that the original value of the mountain bike was 5000 CHF.

- b) 12.5 years

Hint:

- Complete depreciation at time t means $B(t) = 0$.

- 4.7 a) item A: 12.5 years
 item B: 7.5 years

- b) 2.5 years

- 4.8 a) 2nd statement

- b) 1st statement

- c) 3rd statement