Exercises 1 Number sets N, Z, Q, R, set operations

Objectives

- know the definition and elements of natural numbers, integers, rational numbers, and real numbers.
- know and understand what a set, an element of a set, an empty set, a subset, an intersection, a union, and a set
- difference are.
- be able to perform basic set operations.

Problems

1.1 Decide whether each statement is true or false:

a)	$4 \in \mathbb{N}$	b)	$-\frac{14}{7} \in \mathbb{Z}$	c)	$\sqrt{2} \in \mathbb{Q}$
d)	$\sqrt{9} \in \mathbb{N}$	e)	$\sqrt{9} \in \mathbb{Q}$	f)	$\sqrt{9} \in \mathbb{R}$
g)	$1.67854 \in \mathbb{Q}$	h)	$1.67\overline{854} \in \mathbb{Q}$	i)	$\mathbb{N} \subset \mathbb{Z}$
j)	$\mathbb{Z} \subset \mathbb{Q}$	k)	$\mathbb{Q} \subset \mathbb{R}$	1)	$\mathbb{R} \setminus \mathbb{Z} = \mathbb{N}$

1.2 Determine the following sets:

a)	$\mathbb{Z} \setminus \mathbb{N}$	b)	$\mathbb{Z} \cup \mathbb{N}$	c)	$\mathbb{Z}\cap\mathbb{N}$
d)	$\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$	e)	$\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$	f)	$(\mathbb{Q}\setminus\mathbb{Z})\cap\mathbb{N}$

1.3 Look at the sets A, B, and C:

- A = Set of all cities of the world
- B = Set of all European cities
- C = Set of all coastal cities of the world

Find at least five elements of the following sets:

a)	$B \cap C$	b)	$\mathbf{B} \setminus \mathbf{C}$
c)	$\mathbf{C} \setminus \mathbf{B}$	d)	$A \setminus (B \cup C)$

1.4 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.

a)

$$N \cup \mathbb{Z} = \mathbb{Q}$$

$$\mathbb{Q} \setminus \mathbb{Z} = \mathbb{N}$$

$$\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$$

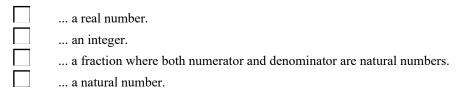
$$\mathbb{Z} \setminus \mathbb{N} = \{-1, -2, -3, ...\}$$

b) A = Set of all cities of the world B = Set of all European cities

$A \cap B = A$
$\mathbf{A} \cup \mathbf{B} = \mathbf{B}$
$\mathbf{B} \in \mathbf{A}$
$\mathbf{B} \subset \mathbf{A}$
$B \subset A$

c) (see next page)

c) Assume that x is a rational number. Therefore, it can be concluded that x is ...



Answers

1.1	a)	true	b)	true	c)	false
	d)	true	e)	true	f)	true
	g)	true	h)	true	i)	true
	j)	true	k)	true	1)	false

1.2 a)
$$\mathbb{Z} \setminus \mathbb{N} = \{0, -1, -2, -3, ...\}$$

- b) $\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$
- c) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$
- d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q}) = \{\}$
- e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$
- f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N} = \{\}$
- 1.3 a) $B \cap C = \{Lisbon, Copenhagen, Barcelona, Naples, Stockholm, ...\}$
 - b) $B \setminus C = \{London, Paris, Madrid, Berlin, Rome, ...\}$
 - c) $C \setminus B = \{Tokyo, San Francisco, Sydney, Rio de Janeiro, Cape Town, ...\}$
 - d) $A \setminus (B \cup C) = \{Chicago, Mexico City, Nairobi, Beijing, Bogotá, ...\}$
- 1.4 a) 3rd statement
 - b) 4th statement
 - c) 1st statement